MEI CORE 2 JAN 2006

1) $140^\circ = k\pi$ radians

$180^\circ = \pi$ radians

$\therefore 140^\circ = \frac{140}{180}\pi$ radians

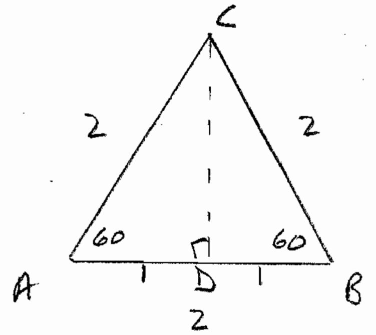
$k = \frac{7}{9}$

2) $\sum_{k=2}^{5} k^3 = 2^3 + 3^3 + 4^3 + 5^3$

$= 8 + 27 + 64 + 125$

$= 224$

3)



3 sides the same

$\therefore \triangle$ is equilateral and each angle is 60°

A perpendicular from C to AB bisects AB at D. $\therefore$ AD = 1

By Pythagoras $CD^2 = AC^2 - AD^2$

$CD^2 = 4 - 1$

$\Rightarrow CD = \sqrt{3}$

$\sin 60^\circ = \frac{CD}{AC} = \frac{\sqrt{3}}{2}$

4)

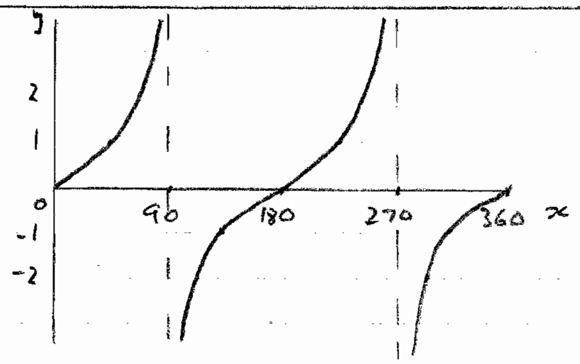
$T_6 = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5) + y_6\right]$

$= \frac{0.5}{2}\left[8.2 + 2(6.4 + 5.5 + 5.0 + 4.7 + 4.4) + 4.2\right]$

$T_6 = 16.1$ units^2

This is an overestimate since top edges of trapeziums would be above the concave curve.

5) i)



5 ii)

$4\sin x = 3\cos x$

$\frac{4\sin x}{\cos x} = 3$

$\frac{\sin x}{\cos x} = \frac{3}{4}$

$\tan x = \frac{3}{4}$

$x = \tan^{-1}\left(\frac{3}{4}\right)$

$x = 36.9^\circ, 216.9^\circ$

for $0^\circ \le x \le 360^\circ$

6) $\frac{dy}{dx} = x^2 - 6x + 9$

$\frac{d^2y}{dx^2} = 2x - 6$

When $x = 3$

$\frac{dy}{dx} = 3^2 - 6(3) + 9$
$= 9 - 18 + 9 = 0$

$\therefore$ a st pt at $x = 3$

When $x = 3$ $\frac{d^2y}{dx^2} = 2(3) - 6 = 0$

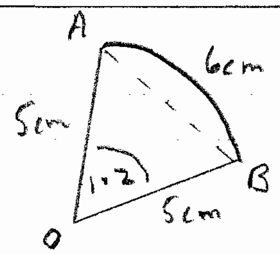
but this does not confirm nature of stationary point

When $x = 2.9$
$\frac{dy}{dx} = 2.9^2 - 6 \times 2.9 + 9 = 0.01$
when $x = 3.1$
$\frac{dy}{dx} = 3.1^2 - 6 \times 3.1 + 9 = 0.01$

As x increases through 3
$\frac{dy}{dx}$ is +ve 0 +ve

$\therefore$ a stationary point of inflection

7)



i) Arc length $= r\theta$

$\Rightarrow 6 = 1.2r$

$\Rightarrow r = \frac{6}{1.2} = 5\text{cm}$

7ii) Cosine Rule

$|AB|^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \cos 1.2$

$\Rightarrow |AB| = 5.65\text{cm}$ to 3 sig fig

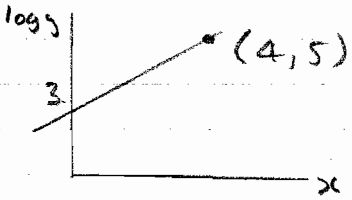
8) $\int \left(x^{\frac{1}{2}} + \frac{6}{x^3} \right) dx$

$= \frac{x^{3/2}}{\frac{3}{2}} - \frac{6}{2x^2} + c$

$= \frac{2x^{3/2}}{3} - \frac{3}{x^2} + c$

9)

i)



Passing through (0,3) and (4,5)

$\frac{\log y - 5}{5 - 3} = \frac{x - 4}{4 - 0}$

$\frac{\log y - 5}{2} = \frac{x - 4}{4}$

$\log y - 5 = \frac{2(x-4)}{4}$

$\log y - 5 = \frac{1}{2}x - 2$

$\log_{10} y = \frac{1}{2}x + 3$

9 ii) $\log_{10} y = \frac{1}{2}x + 3$

$$10^{\log_{10} y} = 10^{\left(\frac{x}{2}+3\right)}$$

$$y = 10^{\left(\frac{x}{2}+3\right)}$$

or $y = 1000 \times 10^{\frac{x}{2}}$

Section B

10) $y = 7 + 6x - x^2$

i) $\frac{dy}{dx} = 6 - 2x$

At t.p $\frac{dy}{dx} = 0$

$$6 - 2x = 0$$

$$6 = 2x$$

$$x = 3$$

When $x = 3$, $y = 7 + 6(3) - (3)^2$

$$y = 7 + 18 - 9 = 16$$

Turning point at $(3, 16)$

Meets y-axis at $(0, 7)$

Meets x-axis when

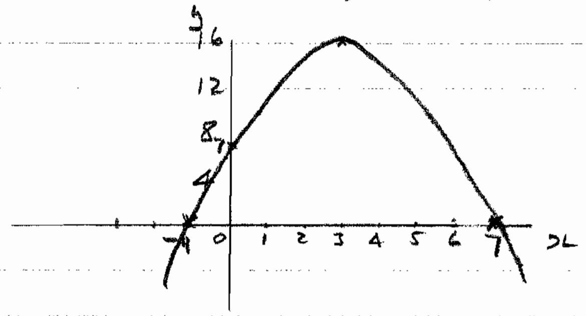
$$7 + 6x - x^2 = 0$$

$$\Rightarrow x^2 - 6x - 7 = 0$$

$$(x-7)(x+1) = 0$$

$$\Rightarrow x = 7 \text{ or } x = -1$$

Cuts x-axis at $(7,0)$ and $(-1,0)$



10 ii) $\int_1^5 (7 + 6x - x^2)\,dx$

$$= \left[7x + 3x^2 - \frac{x^3}{3}\right]_1^5$$

$$= \left(7(5) + 3(5)^2 - \frac{(5)^3}{3}\right) - \left(7 + 3 - \frac{1}{3}\right)$$

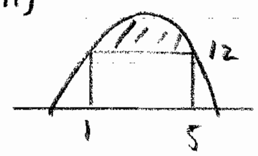
$$= \left(35 + 75 - \frac{125}{3}\right) - \left(10 - \frac{1}{3}\right)$$

$$= 110 - 41\tfrac{2}{3} - 10 + \tfrac{1}{3}$$

$$= 100 - 41\tfrac{1}{3}$$

$$= 58\tfrac{2}{3}$$

10 iii)



Shaded area $= 58\frac{2}{3} - 4 \times 12$

$$= 58\tfrac{2}{3} - 48$$

$$= 10\tfrac{2}{3} \text{ units}^2$$

11) $y = x^3 - 6x + 2$

i) $\frac{dy}{dx} = 3x^2 - 6$

ii) y is decreasing between turning points

At t.p $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6 = 0$

$3x^2 = 6$

$x^2 = 2$

$x = \pm\sqrt{2}$

$\therefore$ turning points at $x = -\sqrt{2}$ and $x = \sqrt{2}$

Function is decreasing for

$-\sqrt{2} < x < \sqrt{2}$

iii) Find gradient at $(-1, 7)$

When $x = -1$, $\frac{dy}{dx} = 3(-1)^2 - 6$

$= -3$

Tgt is line thro $(-1, 7)$ with gradient $= -3$

Using $y - y_1 = m(x - x_1)$

$y - 7 = -3(x - -1)$

$y - 7 = -3(x + 1)$

$y - 7 = -3x - 3$

$y = -3x + 4$

Solve $\begin{cases} y = -3x + 4 \\ y = x^3 - 6x + 2 \end{cases}$

Subst for y

$-3x + 4 = x^3 - 6x + 2$

$x^3 - 6x + 2 + 3x - 4 = 0$

$x^3 - 3x - 2 = 0$

Since line is tgt to curve when $x = -1$, $x = -1$ is a repeated root of this eqn

$(x+1)(x+1) = (x^2 + 2x + 1)$

$$\begin{array}{r|l} & x - 2 \\ \hline x^2+2x+1 & x^3 \qquad\quad - 3x - 2 \\ & x^3 + 2x^2 + x \\ \hline & -2x^2 - 4x - 2 \\ & -2x^2 - 4x - 2 \\ \hline \end{array}$$

$(x+1)(x+1)(x-2) = 0$

Other root $x = 2$

when $x = 2$, $y = -3(2) + 4$

$y = -2$

Tgt crosses curve again at point $(2, -2)$

12) i) A.P. $a = 5$, $d = 2$

A) 10^{th} term $= a + 9d$

$= 5 + 9 \times 2$

$= £23$

B) n^{th} term $= a + (n-1)d$

$a + (n-1)d = 51$

$5 + (n-1)2 = 51$

$(n-1)2 = 46$

$n - 1 = 23$

$n = 24$

He is 24 years old

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{20} = \frac{20}{2}(10 + 19 \times 2)$

$= 10(48)$

$= £480$

12 ii) A) GP $a = 5$, $r = 1.1$

10^{th} term $= ar^9$

$= 5 \times 1.1^9$

$= £11.79$

B) On n^{th} birthday

$ar^{n-1} > 50$

$5 \times 1.1^{n-1} > 50$

$1.1^{n-1} > 10$

$\log_{10} 1.1^{n-1} > \log_{10} 10$

$(n-1)\log_{10} 1.1 > 1$

$(n-1) > \frac{1}{\log_{10} 1.1}$

$n > \frac{1}{\log_{10} 1.1} + 1$

$n > 25.16$

$\therefore n = 26$

when present first exceeds £50