MEI CORE 2 JAN 2009



1) $\int (20x^4 + 6x^{-3/2})\,dx$

$= \dfrac{20x^5}{5} + \dfrac{6x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$= 4x^5 - 12x^{-\frac{1}{2}} + c$

2) $A = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5) + y_6\right]$

$A = \frac{5}{2}\left[4.3 + 2(4.9 + 4.6 + 3.9 + 2.3 + 1.2) + 0\right]$

$A = 95.25$

3) $\displaystyle\sum_{k=1}^{5} \frac{1}{1+k}$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

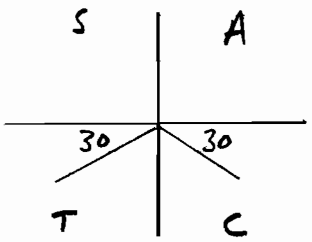
$= \dfrac{30 + 20 + 15 + 12 + 10}{60}$

$= \dfrac{87}{60} = 1\frac{9}{20}$

4) $\sin 2x = -0.5$

for $0 < x < 180°$

$\sin^{-1} 0.5 = 30°$




$2x = 210°, 330°$

$\Rightarrow x = 105°, 165°$

5) See insert

6) AP $a = 7$

$a + 2d = 12$

$\Rightarrow 2d = 5 \Rightarrow d = \frac{5}{2}$

i) $20^{th}$ term $= a + 19d$

$= 7 + 19 \times \frac{5}{2}$

$= 54.5$

ii) Find $S_{50} - S_{20}$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{50} = \frac{50}{2}\left(14 + 49 \times \frac{5}{2}\right)$

$= 3412.5$

$S_{20} = \frac{20}{2}\left(14 + 19 \times \frac{5}{2}\right)$

$= 615$

$S_{50} - S_{20} = 3412.5 - 615$

$= 2{,}797.5$

7) $y = 4x^2 + \frac{1}{x}$

$y = 4x^2 + x^{-1}$

$\Rightarrow \frac{dy}{dx} = 8x - x^{-2}$

$= 8x - \frac{1}{x^2}$

At st. pt. $\frac{dy}{dx} = 0$

$\Rightarrow 8x - \frac{1}{x^2} = 0$

$\Rightarrow 8x^3 - 1 = 0$

$\Rightarrow 8x^3 = 1$

$\Rightarrow x^3 = \frac{1}{8}$

$\Rightarrow x = \sqrt[3]{\frac{1}{8}}$

$\Rightarrow x = \frac{1}{2}$

---

8) $u_1 = 192$, $u_{n+1} = -\frac{1}{2}u_n$

i) This is a GP with $r = -\frac{1}{2}$

$a = 192$, $r = -\frac{1}{2}$

3rd term $= ar^2$

$= 192 \times \left(-\frac{1}{2}\right)^2$

$= 48$

ii) Converges since $\left|-\frac{1}{2}\right| < 1$

$S_\infty = \frac{a}{1-r}$

$= \frac{192}{1 - -\frac{1}{2}}$

$= \frac{192}{3/2} = 128$

---

9) i) $\log_a a = 1$

ii) A) $\log_a x^3 + \log_a \sqrt{x}$

$= 3\log_a x + \log_a x^{\frac{1}{2}}$

$= 3\log_a x + \frac{1}{2}\log_a x$

$= \frac{7}{2}\log_a x$

B) $\log_a\left(\frac{1}{x}\right)$

$= \log_a x^{-1}$

$= -\log_a x$

---

10) i) $y = 7x - x^2 - 6$

$\frac{dy}{dx} = 7 - 2x$

When $x = 2$, $\frac{dy}{dx} = 7 - 2(2) = 3$

When $x = 2$, $y = 7(2) - (2)^2 - 6$

$y = 4$

10i) cont Tgt $y - y_1 = m(x - x_1)$

$$y - 4 = 3(x - 2)$$
$$y - 4 = 3x - 6$$
$$y = 3x - 2$$

10ii) When $x = 1$

$$y = 7(1) - 1^2 - 6$$
$$= 7 - 1 - 6 = 0$$

$\therefore$ crosses $x$-axis at $x = 1$

Solve $7x - x^2 - 6 = 0$

$$\Rightarrow x^2 - 7x + 6 = 0$$
$$(x-6)(x-1) = 0$$
$$\Rightarrow x = 6 \text{ or } x = 1$$

So also crosses $x$-axis at

$$x = 6$$

10iii) $\displaystyle\int_1^2 (7x - x^2 - 6)\,dx$

$$= \left[\frac{7x^2}{2} - \frac{x^3}{3} - 6x\right]_1^2$$

$$= \left(\frac{7(2)^2}{2} - \frac{2^3}{3} - 6(2)\right) - \left(\frac{7(1)^2}{2} - \frac{1^3}{3} - 6(1)\right)$$

$$= \left(14 - \frac{8}{3} - 12\right) - \left(\frac{7}{2} - \frac{1}{3} - 6\right)$$

$$= -\frac{2}{3} + \frac{5}{2} + \frac{1}{3} = \frac{13}{6}$$

Tgt $y = 3x - 2$

Find where it crosses $x$-axis

$$0 = 3x - 2$$
$$\Rightarrow x = \frac{2}{3}$$

Area under tgt between $x = \frac{2}{3}$ and $x = 2$

$= \frac{1}{2}$ base $\times$ height $= \frac{1}{2}\left(2 - \frac{2}{3}\right) \times 4$

$$= \frac{8}{3}$$

Shaded area

= Area under tgt − Area under curve

$$= \frac{8}{3} - \frac{13}{6}$$

$= \frac{1}{2}$ unit$^2$

11i) A) Arc $= r\theta$

$$= 60 \times 2.5$$
$$= 150 \text{ cm}$$

B) Area of sector $= \frac{1}{2}r^2\theta$

Area of surface $= \frac{1}{2}OB^2\theta - \frac{1}{2}OC^2\theta$

$= \frac{1}{2}(140^2 - 60^2) \times 2.5 = 20{,}000 \text{ cm}^2$

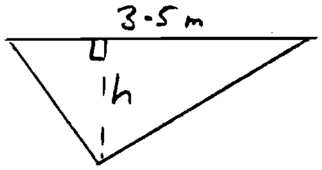
11 ii) Cosine rule

A) $\cos F = \dfrac{p^2 + e^2 - f^2}{2pe}$

$\cos F = \dfrac{3.5^2 + 2.8^2 - 1.6^2}{2 \times 3.5 \times 2.8}$

$\angle EFP = 26.6^\circ$

B)




Area $= \frac{1}{2} \times 3.5 \times h$

but area also

$\frac{1}{2} \times 3.5 \times 2.8 \sin 26.57^\circ$

$= 2.1917 \text{ m}^2$

$\therefore \frac{1}{2} \times 3.5 \times h = 2.1917$

$h = \dfrac{2 \times 2.1917}{3.5}$

$h = 1.25\text{m}$

Shortest distance = 1.25 m

12) i) $y = ab^t$

$\Rightarrow \log_{10} y = \log_{10}(ab^t)$

$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^t$

$\Rightarrow \log_{10} y = \log_{10} a + t \log_{10} b$

12ii) See insert

12iii) From graph $\log_{10} a = 2.31$

$\Rightarrow a = 204$

$\log_{10} b = \text{gradient} = \dfrac{2.72 - 2.31}{5 - 0}$

$\log_{10} b = 0.082 \Rightarrow b = 1.2$

12iv) $y = 204 \times 1.2^t$

At $t = 0$, $y = 204 \times 1.2^0$

$y = 204$

First costing was £204 million

12v) $1000 = 204 \times 1.2^t$

$\dfrac{1000}{204} = 1.2^t$

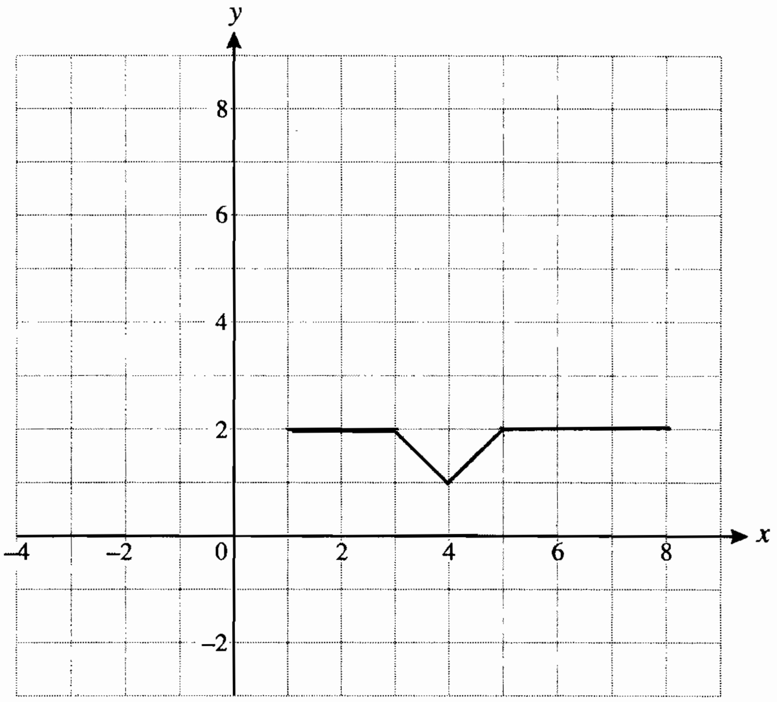
$\log_{10}\left(\dfrac{1000}{204}\right) = \log_{10} 1.2^t$

$\log_{10}\left(\dfrac{1000}{204}\right) = t \log_{10} 1.2$

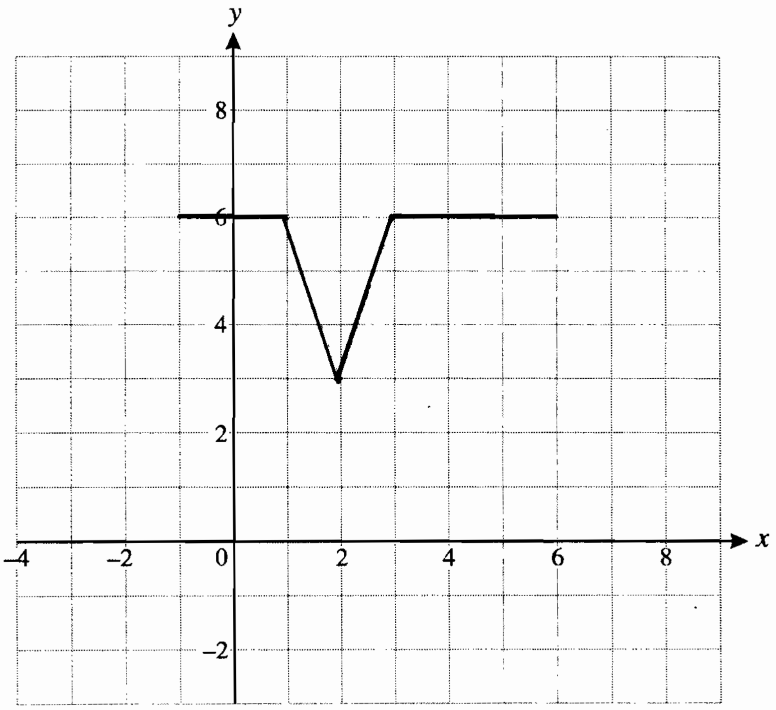
$t = \dfrac{\log_{10}\left(\frac{1000}{204}\right)}{\log_{10} 1.2}$

$t = 8.7$ years

**5** **(i)** $y = \mathrm{f}(x-2)$




**(ii)** $y = 3\mathrm{f}(x)$



12 (ii)

| Years after proposal accepted ($t$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cost (£$y$ million) | 250 | 300 | 360 | 440 | 530 |
| $\log_{10} y$ | 2.398 | 2.477 | 2.556 | 2.643 | 2.724 |