1) $\displaystyle\int\left(x - \frac{3}{x^2}\right)dx$

$= \displaystyle\int\left(x - 3x^{-2}\right)dx$

$= \dfrac{x^2}{2} - \dfrac{3x^{-1}}{-1} + c$

$= \dfrac{x^2}{2} + \dfrac{3}{x} + c$

2) $\underbrace{1\ 3\ 5\ 3}\ \underbrace{1\ 3\ 5\ 3}\ 1\ 3 \ldots$

i) Periodic with period 4

13×4 gives 52 terms

55^{th} will be third in next block of four

55^{th} term $= 5$

ii) $1 + 3 + 5 + 3 = 12$

$13 \times 12 = 156$

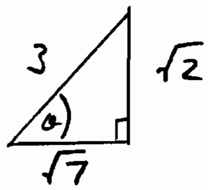
(since 13 complete periods)

Then $1 + 3 + 5 = 9$

Sum of first 55 terms

$= 156 + 9 = 165$

3) $\sin\theta = \dfrac{\sqrt{2}}{3}$



$\tan\theta = \sqrt{\dfrac{2}{7}}$

or $\dfrac{\sqrt{2}}{\sqrt{7}}$

4) Sector Area $= \frac{1}{2}r^2\theta$

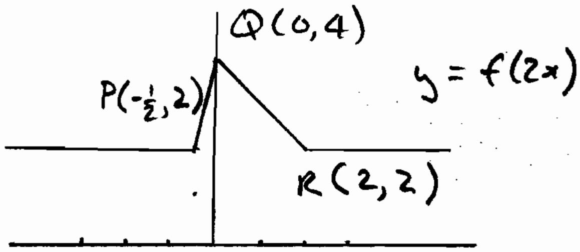
$8.45 = \frac{1}{2}r^2 \times 0.4$

$\dfrac{2 \times 8.45}{0.4} = r^2$

$\Rightarrow \quad r = 6.5$ cm

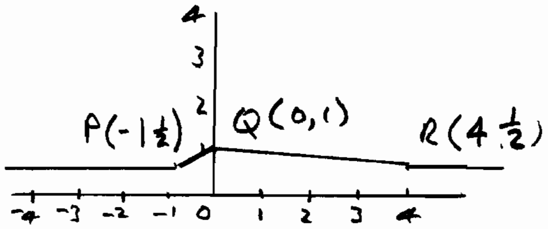
5)

i)



$P(-\frac{1}{2},2)$ $Q(0,4)$ $R(2,2)$

ii)



$P(-1,\frac{1}{2})$ $Q(0,1)$ $R(4,\frac{1}{2})$

6) i) $u_1 = 5$

$u_{n+1} = u_n + 4$

AP $a = 5$, $d = 4$

51st term $= a + 50d$

$= 5 + 50 \times 4 = 205$

6ii) 5 2 0.8

GP $a = 5$ $r = \frac{2}{5}$

$$S_\infty = \frac{a}{1-r}$$

$$S_\infty = \frac{5}{1 - 2/5}$$

$$= \frac{5}{3/5}$$

$$= \frac{25}{3} = 8\frac{1}{3}$$

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7) i) Find $\angle BAD$

Sine rule

$$\frac{5.6}{\sin A} = \frac{8.4}{\sin 79^\circ}$$

$$5.6 \sin 79^\circ = 8.4 \sin A$$

$$\sin A = \frac{5.6 \sin 79^\circ}{8.4}$$

$$A = \sin^{-1}\left(\frac{5.6 \sin 79^\circ}{8.4}\right)$$

$$\angle BAD = 40.9^\circ$$

7ii) $\angle BDC = 180 - 79 = 101^\circ$

Cosine rule

$$BC^2 = 5.6^2 + 7.8^2 - 2 \times 5.6 \times 7.8 \cos 101^\circ$$

$$BC = 10.4 \text{ cm}$$

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8) $y = 6\sqrt{x}$

$$y = 6x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 3x^{-\frac{1}{2}} = \frac{3}{\sqrt{x}}$$

When $x = 16$ $\frac{dy}{dx} = \frac{3}{\sqrt{16}} = \frac{3}{4}$

When $x = 16$ $y = 6\sqrt{16} = 24$

Require line through $(16, 24)$ with gradient $\frac{3}{4}$

$$y - y_1 = m(x - x_1)$$

$$y - 24 = \frac{3}{4}(x - 16)$$

$$y - 24 = \frac{3}{4}x - 12$$

$$y = \frac{3}{4}x + 12$$

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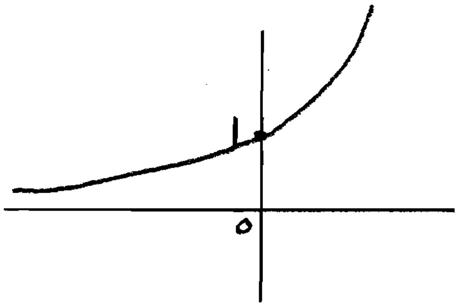
9) i)



ii) $3^{2x+1} = 10$

$$\log_{10} 3^{2x+1} = \log_{10} 10 = 1$$

$$(2x+1)\log_{10} 3 = 1$$

$$2x + 1 = \frac{1}{\log_{10} 3} = 2.0959$$

$2x = 1.0959 \Rightarrow x = 0.55$ to 2dp.

10) i)

$$\frac{d}{dx}\left(x^3 - 3x^2 - 9x\right) = 3x^2 - 6x - 9$$

At st. pt $\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$

$\Rightarrow x = 3$ or $x = -1$

When $x = 3$, $y = 3^3 - 3(3)^2 - 9(3) = -27$

When $x = -1$, $y = (-1)^3 - 3(-1)^2 - 9(-1) = 5$

$$\frac{d^2y}{dx^2} = 6x - 6$$

When $x = 3$, $\frac{d^2y}{dx^2} = 18 - 6 = 12$

$\frac{d^2y}{dx^2} > 0 \therefore$ a min

When $x = -1$ $\frac{d^2y}{dx^2} = -6 - 6 = -12$

$\frac{d^2y}{dx^2} < 0 \therefore$ a max

Max at $(-1, 5)$

Min at $(3, -27)$

(Note: y coords were not reqd.)

10 ii) $x^3 - 3x^2 - 9x = 0$

$\Rightarrow x(x^2 - 3x - 9) = 0$

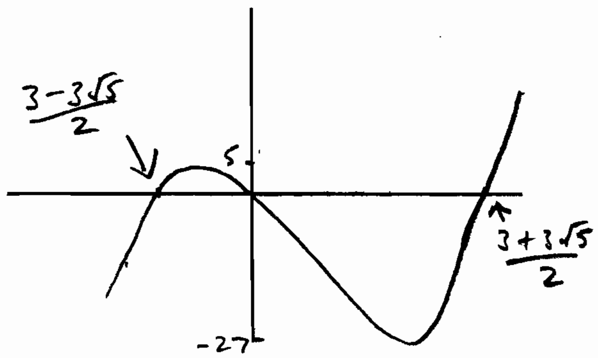
Either $x = 0$ or $x^2 - 3x - 9 = 0$

$$x = \frac{3 \pm \sqrt{9 + 36}}{2}$$

$$x = \frac{3 \pm \sqrt{45}}{2} = \frac{3 \pm 3\sqrt{5}}{2}$$

Crosses x-axis at

$(0,0)$ $\left(\frac{3+3\sqrt{5}}{2}, 0\right)$ $\left(\frac{3-3\sqrt{5}}{2}, 0\right)$



11 i)

$$A \approx \frac{1.5}{2}\left[2.3 + 2(2.7 + 3.3 + 4 + 4.8 + 5.2 + 5.2 + 4.4) + 2\right]$$

$A \approx 47.625$ units^2

11 ii)

$$1.5\left[2.3 + 2.7 + 3.3 + 4.0 + 4.8 + 5.2 + 4.4 + 2.0\right]$$

Lower bound $= 43.05$

11 iii) $y = -0.013x^3 + 0.16x^2 - 0.082x + 2.4$

$$A = \int_0^{12} y \, dx$$

$$= \left[-0.013\frac{x^4}{4} + 0.16\frac{x^3}{3} - 0.082\frac{x^2}{2} + 2.4x \right]_0^{12}$$

$$= 47.664$$

11iv) When $x = 7.5$ m

$y = 5.3$ m

This is 0.1 m higher than measured value. Approx 2% higher so quite a good fit.

12 i) $$P = a \times 10^{bt}$$

$$\Rightarrow \log_{10} P = \log_{10}\left(a \times 10^{bt}\right)$$

$$\Rightarrow \log_{10} P = \log_{10} a + \log_{10} 10^{bt}$$

$$\Rightarrow \log_{10} P = \log_{10} a + bt \log_{10} 10$$

$$\Rightarrow \log_{10} P = \log_{10} a + bt$$

$$y = c + mx$$

compares with straight line

$\log_{10} a$ is vertical intercept

b is gradient

12ii) See insert

12iii) Points on line of best fit

$(60, 2.69)$ $(0, 1.99)$

$$b = \frac{2.69 - 1.99}{60 - 0} = \frac{0.70}{60}$$

$$b = 0.01167$$

From graph intercept

$$\log_{10} a = 1.99$$

$$\Rightarrow a = 97.72$$

$$\therefore P = 97.72 \times 10^{0.01167t}$$

12iv) In 2050 $t = 105$

$$P = 97.72 \times 10^{0.01167 \times 105}$$

$$P = 1641.85$$

Population 1642 million

Estimate is unreliable when extrapolating well outside the data set.

12 (ii)

Year	1955	1965	1975	1985	1995	2005
t	10	20	30	40	50	60
P	131	161	209	277	372	492
$\log_{10} P$	2.12	2.21	2.32	2.44	2.57	2.69