MEI CORE 2 MAY 2010

1) $u_1 = 1, \quad u_{n+1} = \dfrac{u_n}{1+u_n}$

$u_2 = \dfrac{1}{1+1} = \dfrac{1}{2}$

$u_3 = \dfrac{\frac{1}{2}}{1+\frac{1}{2}} = \dfrac{\frac{1}{2}}{3/2} = \dfrac{1}{3}$

$u_4 = \dfrac{\frac{1}{3}}{1+\frac{1}{3}} = \dfrac{\frac{1}{3}}{4/3} = \dfrac{1}{4}$

2) i) $\displaystyle\sum_{r=2}^{5} \frac{1}{r-1}$

$= \dfrac{1}{2-1} + \dfrac{1}{3-1} + \dfrac{1}{4-1} + \dfrac{1}{5-1}$

$= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$= 2\frac{1}{12}$

2ii) $2\times3 + 3\times4 + 4\times5 + 5\times6 + 6\times7$

$= \displaystyle\sum_{r=2}^{6} r(r+1)$

3) i) $\dfrac{d}{dx}\left(x^3 - 6x^2 - 15x + 50\right)$

$= 3x^2 - 12x - 15$

3 ii) At st.pt. $\dfrac{dy}{dx} = 0$

$\Rightarrow \quad 3x^2 - 12x - 15 = 0$

$3(x^2 - 4x - 5) = 0$

$3(x-5)(x+1) = 0$

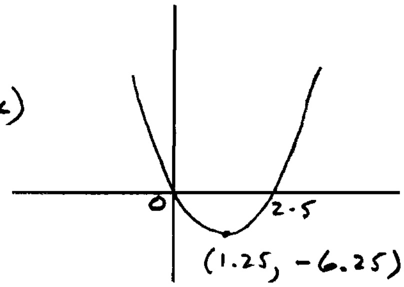
$\Rightarrow \quad x = 5 \quad \text{or} \quad x = -1$

x coords of st. pts.

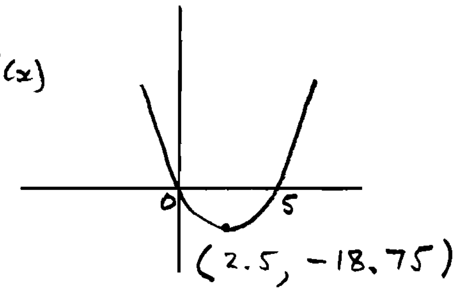
$x = 5, \quad x = -1$

4) $f(x) = x^2 - 5x$

$y = f(2x)$



$y = 3f(x)$



5) $\displaystyle\int_2^5 \left(1 - \frac{6}{x^3}\right) dx$

$= \displaystyle\int_2^5 \left(1 - 6x^{-3}\right) dx$

$= \left[x - \dfrac{6x^{-2}}{-2} \right]_2^5$

$= \left[x + \dfrac{3}{x^2} \right]_2^5$

$= \left(5 + \dfrac{3}{25}\right) - \left(2 + \dfrac{3}{4}\right) = 2.37$

6) $\frac{dy}{dx} = 6x^2 + 12x^{\frac{1}{2}}$

$\Rightarrow y = \frac{6x^3}{3} + \frac{12x^{3/2}}{3/2} + c$

$\Rightarrow y = 2x^3 + 8x^{3/2} + c$

Passes through $(4, 10)$ so

$10 = 2 \times 4^3 + 8 \times 4^{3/2} + c$

$10 = 128 + 64 + c$

$10 - 128 - 64 = c$

$-182 = c$

$\therefore y = 2x^3 + 8x^{3/2} - 182$

7) $\log_a x^3 + \log_a \sqrt{x}$

$= 3\log_a x + \log_a x^{\frac{1}{2}}$

$= 3\log_a x + \frac{1}{2}\log_a x$

$= \frac{7}{2}\log_a x$

8) $4\sin^2\theta = 3 + \cos^2\theta$

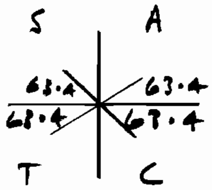
$4\sin^2\theta = 3 + 1 - \sin^2\theta$

$5\sin^2\theta = 4$

$\sin^2\theta = \frac{4}{5}$

$\sin\theta = \pm\sqrt{\frac{4}{5}}$

$\theta = 63.4°$



$\theta = 63.4°, 116.6°, 243.4°, 296.6°$

9) $y = ax^n$

$(2, 6)$ and $(3, 18)$ on curve

$\Rightarrow 6 = a \times 2^n$ ①

and $18 = a \times 3^n$ ②

② ÷ ① $\frac{18}{6} = \frac{a \times 3^n}{a \times 2^n}$

$3 = \left(\frac{3}{2}\right)^n$

$\log_{10} 3 = n \log_{10}\left(\frac{3}{2}\right)$

$n = \frac{\log_{10} 3}{\log_{10}\left(\frac{3}{2}\right)} = 2.71$ to 2 d.p.

Subst for n in ①

$6 = a \times 2^{2.709511291}$

$\frac{6}{2^{2.709511291}} = a$

$a = 0.92$ to 2dp

$n = 2.71$ to 2 d.p

10) i)

$$y = x^4$$

$$\Rightarrow \frac{dy}{dx} = 4x^3$$

when $x = 2$, $\frac{dy}{dx} = 4 \times 2^3 = 32$

when $x = 2$, $y = 2^4 = 16$

tgt has gradient $= 32$
and passes thro $(2, 16)$

$$y - y_1 = m(x - x_1)$$

$$y - 16 = 32(x - 2)$$

$$y - 16 = 32x - 64$$

$$y = 32x - 48$$

10ii)

When $x = 2.1$
$y = 2.1^4 = 19.4481$

Gradient of chord between $(2, 16)$ and $(2.1, 19.4481)$

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{19.4481 - 16}{2.1 - 2}$$

$$= 34.481$$

10iii) A)

$$(2+h)^4$$

$$= 2^4 + 4 \times 2^3 h + 6 \times 2^2 h^2 + 4 \times 2 h^3 + h^4$$

$$= 16 + 32h + 24h^2 + 8h^3 + h^4$$

B)

$$\frac{(2+h)^4 - 2^4}{h}$$

$$= \frac{32h + 24h^2 + 8h^3 + h^4}{h}$$

$$= 32 + 24h + 8h^2 + h^3$$

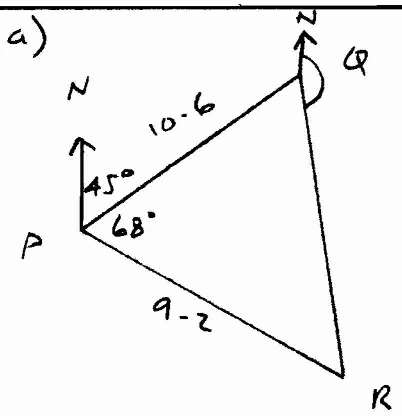
c)

Gradient of curve = gradient of tangent at $x = 2$ which is obtained by letting $h \to 0$

Then gradient of chord becomes gradient of tangent

$$= 32 + 0 + 0 + 0 = 32$$

11) a)



Find QR using cosine rule

$$QR^2 = 10.6^2 + 9.2^2 - 2 \times 10.6 \times 9.2 \cos 68°$$

$$QR^2 = 123.93673$$

$$QR = 11.1 \text{ km}$$

First find $\angle PQR$ using sine rule

$$\frac{9.2}{\sin(\angle PQR)} = \frac{11.13}{\sin 68°}$$

$$9.2 \sin 68 = 11.13 \sin(\angle PQR)$$

$$\frac{9.2 \sin 68}{11.13} = \sin(\angle PQR)$$

$$\angle PQR = 50.0°$$

11 a) cont)

Bearing of R from Q

$= 360° - 135° - 50°$

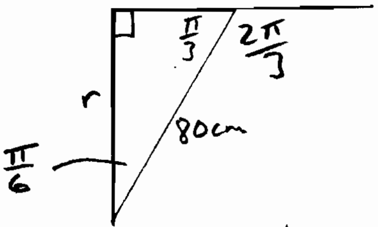
$= 175°$

11b) i) Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 80^2 \times \frac{2\pi}{3}$

$= 6702 \text{ cm}^2$

ii)



$\sin\left(\frac{\pi}{3}\right) = \frac{r}{80}$

$\frac{\sqrt{3}}{2} = \frac{r}{80}$

$80\sqrt{3} = 2r$

$40\sqrt{3} = r$

$r = 40\sqrt{3}$ cm

Area of ΔCDA using $\frac{1}{2}ab\sin C$

$= \frac{1}{2} \times r \times 80 \sin\frac{\pi}{6}$

$= \frac{1}{2} \times 40\sqrt{3} \times 80 \times \frac{1}{2}$

$= 800\sqrt{3}$

$= 1386 \text{ cm}^2$

iii) Area of cross-section of rudder

= Area of ΔCDA + Area of sector ABC − Area of quarter circle DCE

$= 6702 + 1386 - \frac{\pi r^2}{4}$

$= 6702 + 1386 - \frac{\pi \times (40\sqrt{3})^2}{4}$

$= 4318 \text{ cm}^2$

12) i) A)

Stage	Nodes
1	1
2	2
3	4
n	2^{n-1}

10^{th} stage $= 2^9$ nodes $= 512$

Buds $= 512 \times 2 = 1024$

B) Stems

$1 + 2 + 4 + 8$

GP $a = 1$, $r = 2$

$S_{10} = \frac{a(r^n - 1)}{r - 1} = \frac{1(2^{10} - 1)}{2 - 1}$

$= 1023$

12ii) A) Leaves at each stage

7, 14, 28, 56

GP $a = 7$, $r = 2$

12ii) A cont)

Total leaves on n stage plant

$7 + 14 + 28 + 56 + \dots +$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$= \frac{7(2^n - 1)}{2-1} = 7(2^n - 1)$$

12ii) B)

$7(2^n - 1) > 200000$

$7 \times 2^n - 7 > 200000$

$7 \times 2^n > 200007$

$\log_{10}(7 \times 2^n) > \log_{10} 200007$

$\log_{10} 7 + \log_{10}(2^n) > \log_{10} 200007$

$n \log_{10} 2 > \log_{10} 200007 - \log_{10} 7$

$$n > \frac{\log_{10} 200007 - \log_{10} 7}{\log_{10} 2}$$

$n > 14.8$

minimum value of $n = 15$