1) $\frac{7\pi}{6}$ radians $= \frac{7\pi}{6} \times \frac{180}{\pi}^{\circ}$

$= 210^{\circ}$

2) GP $a = 5.4$ $r = 0.1$

i) 4^{th} term $= ar^3 = 5.4 \times 0.1^3$

$= 0.0054$

ii) $S_\infty = \frac{a}{1-r} = \frac{5.4}{1-0.1}$

$S_\infty = 6$

3) Let $f(x) = x^2 + 5$

then $3f(x) = 3x^2 + 15$

A one-way stretch parallel to the y-axis by a scale factor of 3

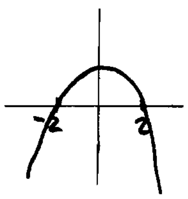
4) $f(x) = 12x - x^3$

$\Rightarrow f'(x) = 12 - 3x^2$

f is an increasing function when $f'(x) > 0$

$\Rightarrow 12 - 3x^2 > 0$
$3(4 - x^2) > 0$

$3(2-x)(2+x) > 0$



$f(x)$ is increasing for $-2 < x < 2$

5) i) $B(3.1, 2^{3.1})$

$A(3, 2^3)$

Gradient of chord AB

$= \frac{2^{3.1} - 2^3}{3.1 - 3} = 5.7419$

$= 5.74$ to 2 d.p.

ii) Use $C(3.05, 2^{3.05})$

Gradient of chord AC

$= \frac{2^{3.05} - 2^3}{3.05 - 3} = 5.6424$

$= 5.64$ to 2 d.p.

(Any point between 3 and 3.1 would be suitable as C)

6) $\frac{dy}{dx} = 6\sqrt{x} = 6x^{\frac{1}{2}}$

$\Rightarrow y = \frac{6x^{3/2}}{3/2} + c$

$y = 4x^{3/2} + c$

Passes through $(9, 105)$

so $105 = 4 \times 9^{3/2} + c$

$105 = 4 \times 27 + c$

$105 = 108 + c$

$\Rightarrow c = -3$

$\therefore y = 4x^{3/2} - 3$

7) $r = 6\text{cm}$, $\theta = 1.6$ radians

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 6^2 \times 1.6 = 28.8 \text{ cm}^2$

Area of segment =

Area of sector − Area of $\triangle$

$= 28.8 - \frac{1}{2} r \times r \times \sin\theta$

$= 28.8 - \frac{1}{2} \times 6 \times 6 \times \sin 1.6$

$= 10.8 \text{ cm}^2$ to 3 s.f.

8) AP 11^{th} term $= a + 10d = 1$ ①

$S_{10} = \frac{n}{2}(2a + (n-1)d)$

$= \frac{10}{2}(2a + 9d) = 120$

$\Rightarrow 10a + 45d = 120$ ②

from ① $a = 1 - 10d$

Subst for a in ②

$10(1 - 10d) + 45d = 120$

$10 - 100d + 45d = 120$

$-55d = 110$

$\Rightarrow d = -2$

Also $a = 1 - 10(-2)$

$\Rightarrow a = 21$

4^{th} term $= a + 3d$

$= 21 + 3(-2)$

$= 15$

9) $5^x = 235$

$\log_{10}(5^x) = \log_{10} 235$

$x \log_{10} 5 = \log_{10} 235$

$x = \frac{\log_{10} 235}{\log_{10} 5}$

$x = 3.39$ to 2 d.p.

10) $2\sin^2\theta = \cos\theta + 2$

$2(1 - \cos^2\theta) = \cos\theta + 2$

$2 - 2\cos^2\theta = \cos\theta + 2$

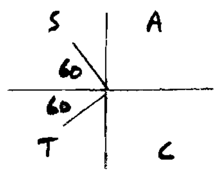
$0 = 2\cos^2\theta + \cos\theta$

$0 = \cos\theta(2\cos\theta + 1)$

$\Rightarrow \cos\theta = 0$ or $\cos\theta = -\frac{1}{2}$

When $\cos\theta = 0$, $\theta = 90°, 270°$

When $\cos\theta = -\frac{1}{2}$, $\theta = 120°, 240°$



Solution for $0 \le \theta < 360°$

$\theta = 90°, 120°, 240°, 270°$

11) i) $f(x) = (x+2)(x+5)(x-2)$

ii)

$(x+2)(x-2) = (x^2-4)$

$(x^2-4)(x+5)$

$= x^3 - 4x + 5x^2 - 20$

$\therefore\ f(x) = x^3 + 5x^2 - 4x - 20$

iii)

$f'(x) = 3x^2 + 10x - 4$

At t.p. $f'(x) = 0$

$\Rightarrow\ 3x^2 + 10x - 4 = 0$

$x = \dfrac{-10 \pm \sqrt{10^2 + 48}}{6}$

$x = \dfrac{-10 \pm \sqrt{148}}{6}$

$x = \dfrac{-10 \pm 12.166}{6}$

$x = 0.361$ or -3.694

From graph $x = 0.361$ refers to min point

$\therefore\ x = 0.4$ to 1 d.p.

Max point when $x = -3.694$

$f(-3.694)$

$= (-3.694)^3 + 5(-3.694)^2 - 4(-3.694) - 20$

$= 12.597$

Max pt at $(-3.7, 12.6)$ to 1 d.p.

11 iv) $y = f(2x)$

represents a stretch parallel to x-axis by scale factor $\frac{1}{2}$

Max at $\left(-\dfrac{3.694}{2}, 12.597\right)$

$= (-1.847, 12.597)$

$= (-1.8, 12.6)$ to 1 d.p.

12)

i) Area $\approx \dfrac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4) + y_5\right]$

$= \dfrac{0.1}{2}\left[0.14 + 2(0.22 + 0.31 + 0.36 + 0.32) + 0.16\right]$

$= 0.136\ m^2$

Volume = Area of cross-section × length

$= 0.136 \times 2.5\ m^3$

Volume approximately $0.34\ m^3$

ii) $\displaystyle\int_0^{0.5} (8x^3 - 3x^2 - 0.5x - 0.15)\,dx$

$= \left[\dfrac{8x^4}{4} - \dfrac{3x^3}{3} - \dfrac{0.5x^2}{2} - 0.15x\right]_0^{0.5}$

$= \left[2x^4 - x^3 - 0.25x^2 - 0.15x\right]_0^{0.5}$

$= (2 \times 0.5^4 - 0.5^3 - 0.25(0.5)^2 - 0.15 \times 0.5)$

$- (0 - 0 - 0 - 0)$

$= -0.1375$

12ii) cont) -0.1375 represents the shaded area between the curve and the x-axis

The $-$ sign indicates the area is below the x-axis

Model estimates cross-section to have area $= 0.1375\ m^2$

$\therefore$ volume estimate

$= 0.1375 \times 2.5 = 0.34375 m^3$

---

13) i)

$$P = at^b$$

$$\Rightarrow \quad \log_{10} P = \log_{10}(at^b)$$

$$\Rightarrow \quad \log_{10} P = \log_{10} a + \log_{10} t^b$$

$$\Rightarrow \log_{10} P = \log_{10} a + b \log_{10} t$$

Plotting $\log_{10} P$ against $\log_{10} t$

compares with $y = mx + c$ giving a straight line with gradient b and an intercept of $\log_{10} a$ on the vertical axis.

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ii) See Insert

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iii)

Gradient $b = \dfrac{1.78 - 1.49}{1.3 - 0}$

$b = 0.223$

From graph

$\log_{10} a = 1.49$

$\Rightarrow a = 10^{1.49} = 30.9 \approx 31$

Eqn is $P = 31t^{0.223}$

---

iv)

When $t = 22$

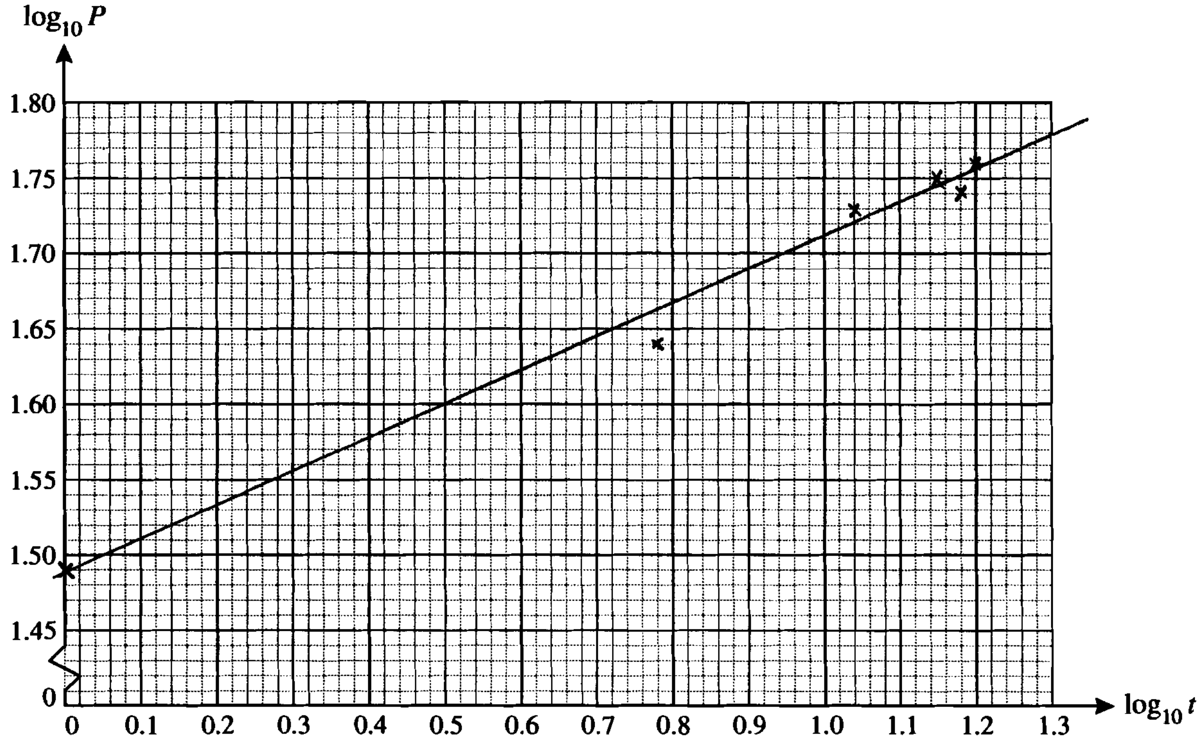
$P = 31 \times 22^{0.223}$

$P = 61.76\ \%$

$P \approx 62\ \%$

13 (ii)

Year	1986/87	1991/92	1996/97	1999/00	2000/01	2001/02
t	1	6	11	14	15	16
P	31	44	54	56	55	57
$\log_{10} t$	0	0.78	1.04	1.15	1.18	1.20
$\log_{10} P$	1.49	1.64	1.73	1.75	1.74	1.76



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