OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4752

Concepts for Advanced Mathematics (C2)

Tuesday **6 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Question **12**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages and an insert.

Section A (36 marks)

1 Write down the values of $\log_a a$ and $\log_a (a^3)$. [2]

2 The first term of a geometric series is 8. The sum to infinity of the series is 10.

Find the common ratio. [3]

3 θ is an acute angle and $\sin\theta = \frac{1}{4}$. Find the exact value of $\tan\theta$. [3]

4 Find $\int_1^2 \left(x^4 - \frac{3}{x^2} + 1\right) \mathrm{d}x$, showing your working. [5]

5 The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 3 - x^2$. The curve passes through the point $(6, 1)$. Find the equation of the curve. [4]

6 A sequence is given by the following.

$$u_1 = 3$$
$$u_{n+1} = u_n + 5$$

(i) Write down the first 4 terms of this sequence. [1]

(ii) Find the sum of the 51st to the 100th terms, inclusive, of the sequence. [4]

7 **(i)** Sketch the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$.

On the same axes, sketch the graph of $y = \cos 2x$ for $0° \leqslant x \leqslant 360°$. Label each graph clearly. [3]

(ii) Solve the equation $\cos 2x = 0.5$ for $0° \leqslant x \leqslant 360°$. [2]

8 Given that $y = 6x^3 + \sqrt{x} + 3$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [5]

9 Use logarithms to solve the equation $5^{3x} = 100$. Give your answer correct to 3 decimal places. [4]

Section B (36 marks)

10 **(i)**

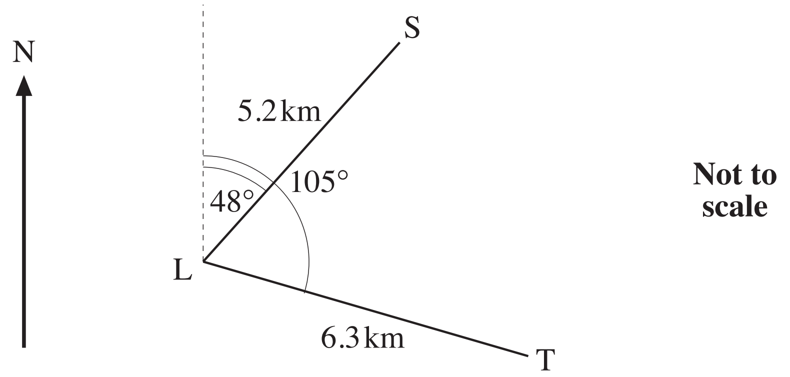

Fig. 10.1

At a certain time, ship S is 5.2 km from lighthouse L on a bearing of 048°. At the same time, ship T is 6.3 km from L on a bearing of 105°, as shown in Fig. 10.1.

For these positions, calculate

(*A*) the distance between ships S and T, [3]

(*B*) the bearing of S from T. [3]

(ii)

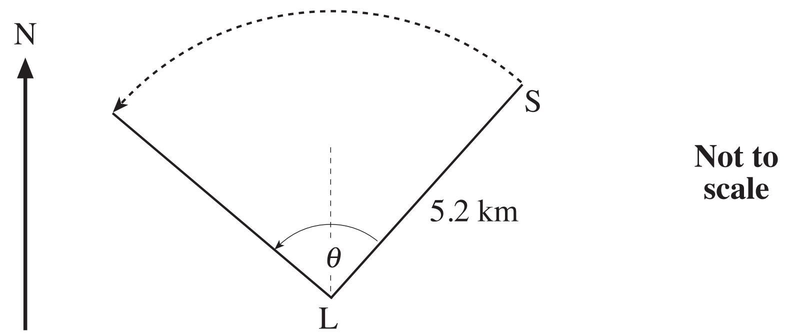

Fig. 10.2

Ship S then travels at 24 km h^{-1} anticlockwise along the arc of a circle, keeping 5.2 km from the lighthouse L, as shown in Fig. 10.2.

Find, in radians, the angle θ that the line LS has turned through in 26 minutes.

Hence find, in degrees, the bearing of ship S from the lighthouse at this time. [5]

[Turn over

11 A cubic curve has equation $y = x^3 - 3x^2 + 1$.

(i) Use calculus to find the coordinates of the turning points on this curve. Determine the nature of these turning points. [5]

(ii) Show that the tangent to the curve at the point where $x = -1$ has gradient 9.

Find the coordinates of the other point, P, on the curve at which the tangent has gradient 9 and find the equation of the normal to the curve at P.

Show that the area of the triangle bounded by the normal at P and the x- and y-axes is 8 square units. [8]

12 **Answer the whole of this question on the insert provided.**

A colony of bats is increasing. The population, P, is modelled by $P = a \times 10^{bt}$, where t is the time in years after 2000.

(i) Show that, according to this model, the graph of $\log_{10} P$ against t should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis. [3]

(ii) The table gives the data for the population from 2001 to 2005.

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10000	11300	12800

Complete the table of values on the insert, and plot $\log_{10} P$ against t. Draw a line of best fit for the data. [3]

(iii) Use your graph to find the equation for P in terms of t. [4]

(iv) Predict the population in 2008 according to this model. [2]

Candidate Name	Centre Number	Candidate Number

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INSERT

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INSTRUCTIONS TO CANDIDATES

- This **insert** should be used in Question **12.**
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 2 printed pages.

12 **(i)** ....

....

....

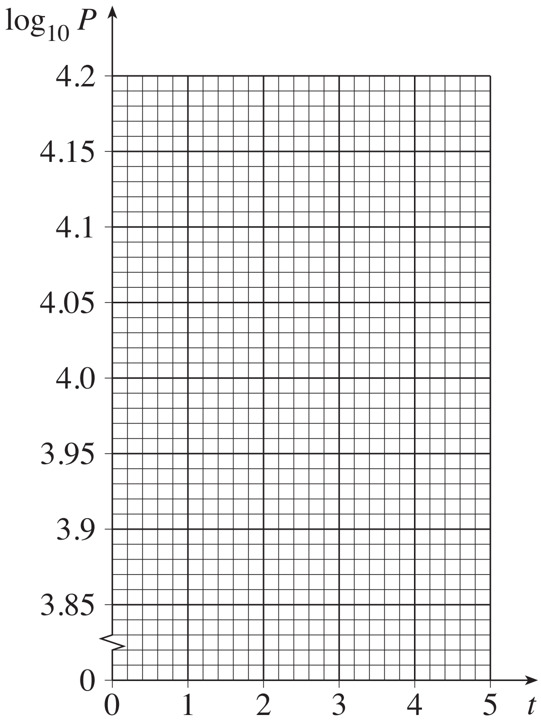
....

....

....

(ii)

Year	2001	2002	2003	2004	2005
t	1	2	3	4	5
P	7900	8800	10 000	11 300	12 800
$\log_{10} P$					



(iii) ....

....

....

(iv) ....

....

Mark Scheme 4752
June 2006

Section A

1	1, 3	1,1		2	
2	$r = 0.2$	3	M1 for $10 = 8/(1 - r)$, then M1 dep't for any correct step	3	
3	$1/\sqrt{15}$ i.s.w. not +/–	3	M2 for $\sqrt{15}$ seen M1 for rt angled triangle with side 1 and hyp 4, or $\cos^2\theta = 1 - 1/4^2$.	3	
4	$x^5/5 - 3x^{-1}/-1 + x$ [value at 2 – value at 1] attempted 5.7 c.a.o.	B3 M1 A1	1 each term dep't on B2	5	
5	[y =] $3x - x^3/3$ $+ c$ subst of (6, 1) in their eqn with c $y = 3x - x^3/3 + 55$ c.a.o	B1 B1 M1 A1	 Dep't on integration attempt Dep't on B0B1 Allow $c = 55$ isw	4	17
6	(i) 3, 8, 13, 18 (ii) use of $n/2[2a + (n-1)d]$ (S_{100} =) 25 050 or (S_{50} =) 6275 (S_{49} =) 6027 or (S_{51} =) 6528 their($S_{100} - S_{50}$) dep't on M1 18 775 cao	B1 M1 A1 M1 A1	Ignore extras Use of $a + (n-1)d$ $u_{51} = 253$ $u_{100} = 498$ $u_{50} = 248$ $u_{52} = 258$ 50/2(their($u_{51} + u_{100}$)) dep't on M1 or 50/2[2 × their(u_{51}) + 49 × 5]	5	
7	(i) sketch of correct shape correct period and amplitude period halved for $y = \cos 2x$; amplitude unchanged (ii) 30, 150, 210, 330	G1 G1 G1 B2	Not ruled lines need 1 and –1 indicated; nos. on horiz axis not needed if one period shown B1 for 2 of these, ignore extras outside range.	5	
8	$\sqrt{x} = x^{\frac{1}{2}}$ soi $18x^2$, $\frac{1}{2}x^{-\frac{1}{2}}$ $36x$ $Ax^{-3/2}$ (from $Bx^{-\frac{1}{2}}$)	B1 B1B1 B1 B1	 –1 if d/dx(3) not = 0 any A,B	5	
9	$3x \log 5 = \log 100$ $3x = \log 100/\log 5$ $x = 0.954$	M1 M1 A2	allow any or no base or $3x = \log_5 100$ dep't A1 for other rot versions of 0.9537... SC B2/4 for 0.954 with no log wkg SC B1 r.o.t. 0.9537...	4	19

Section B

10	**i** (*A*)	$5.2^2 + 6.3^2 - 2\times5.2 \times 6.3 \times\cos$ "57"	M2	M1 for recognisable attempt at cos rule. or greater accuracy		
		ST = 5.6 or 5.57 cao	A1		3	
	i (*B*)	sin T/5.2 = sin(their 57)/their ST	M1	Or sin S/6.3 = … or cosine rule		
		T=51 to 52 or S = 71 to 72	A1			
		bearing 285 + their T or 408 – their S	B1	If outside 0 to 360, must be adjusted	3	
	ii	5.2θ, $24 \times 26/60$	B1B1			
		$\theta = 1.98$ to 2.02	B1	Lost for all working in degrees		
		$\theta =$ their $2 \times 180/\pi$ or 114.6°...	M1	Implied by 57.3		
		Bearing = 293 to 294 cao	A1		5	11
11	**i**	$y' = 3x^2 - 6x$	B1	condone one error		
		use of $y' = 0$	M1			
		(0, 1) or (2, –3)	A2	A1 for one correct or $x = 0$, 2 SC B1 for (0,1) from their y'		
		sign of y'' used to test or y' either side	T1	Dep't on M1 or y either side or clear cubic sketch	5	
	ii	$y'(-1) = 3 + 6 = 9$	B1			
		$3x^2 - 6x = 9$	M1	ft for their y'		
		$x = 3$	A1	implies the M1		
		At P $y = 1$	B1			
		grad normal = –1/9 cao	B1			
		$y - 1 = -1/9\ (x - 3)$	M1	ft their (3, 1) and their grad, not 9		
		intercepts 12 and 4/3or use of $\int_0^{12} \frac{4}{3} - \frac{1}{9}x\ dx$ (their normal)	B1	ft their normal (linear)		
		$\frac{1}{2} \times 12 \times 4/3$ cao	A1		8	13
12	**i**	$\log_{10} P = \log_{10} a + \log_{10}10^{bt}$	B1	condone omission of base		
		$\log_{10} 10^{bt} = bt$	B1			
		intercept indicated as $\log_{10}a$	B1		3	
	ii	3.9(0), 3.94, 4(.00), 4.05, 4.11	T1	to 3 sf or more; condone one error		
		plots ft	P1	1 mm		
		line of best fit ft	L1	ruled and reasonable	3	
	iii	(gradient =) 0.04 to 0.06 seen	M1			
		(intercept =) 3.83 to 3.86 seen	M1			
		(a =) 6760 to 7245 seen	A1			
		$P = 7000 \times 10^{0.05t}$ oe	A1	7000×1.12^t SC $P = 10^{0.05t + 3.85}$ left A2	4	
	iv	17 000 to 18 500	B2	14 000 to 22 000 B1	2	12

4752 - Concepts for Advanced Mathematics (C2)

General Comments

The paper was well received. All questions were accessible but there was wide-spread misunderstanding of the requirements in question 3 and the diagram in question 10. Although good candidates had no trouble completing the paper others could not manage all the work in questions 11 and 12, perhaps due to lack of knowledge, perhaps having spent too much time on earlier problems.
There were many excellent scripts and just a few from candidates totally unprepared at this level.

Comments on Individual Questions

Section A

1) This was often totally correct. Surprisingly some could evaluate the first expression but not the second.

2) This was very well done by the majority. Just a few thought that the sum was a/(r-1). Many could not do the simple algebra needed to solve 10=8/(1-r).

3) It must be made clear to candidates, as part of the preparation for this paper, that a request for 'the exact value' of anything usually implies that calculators must not be used. To put 0.25 into a calculator on one function and take it out on another gives the answer of 0.258 in a matter of a few seconds; but 0.258 is not exact, nor is 0.2581988897. The few who put $\sin\theta = ¼$ onto a sketch of a right angled triangle were usually successful. The few who put it into $\sin^2\theta + \cos^2\theta = 1$ were less successful as this method was more prone to errors such as $\sin\theta + \cos\theta = 1$ at some stage, or, $1-(1/4)^2$ onwards being done by calculator.

4) This was well done by the majority. Some had difficulty integrating the second or third term but the general method for definite integration was well known. Weaker candidates substituted into the function without integrating, or substituted after differentiating.

5) Many knew that this was a cubic curve of the form y=f(x) + c and they successfully calculated the value of c for full marks. Many did not know the process and they worked with $y = mx+c$ with m as $3-6^2 = -33$ or with m = dy/dx to get $y = (3-x^2)x+c$; neither of these methods attracted any marks.

6) Candidates who knew how to work with arithmetic progressions fell into two distinct classes; those who could cope with "51st to 100th inclusive" and those who could not. The former scored 5 marks, the latter only 3 marks. There were very many who calculated $S_{100} - S_{51}$ or who summed the terms starting at the 51st thinking there were only 49 of them. Weaker candidates used incorrect formulae or G.P. formulae or could not understand the suffix notation.

7) (i) The sketches of cos$x$ and cos2x were often essentially correct and received full marks even when they were not things of beauty. We did need an indication that the amplitude was 1. The common errors with cos2x were to double the amplitude or double the period.

(ii) Many correctly found the solutions to the equation, many only two. Some converted cos2x = 0.5 to cos$x$ = 0.25.

8) This was very well done indeed. There were two marks for dealing with $6x^3$. There was a mark for using $\sqrt{x}$ as $x^{1/2}$ and two for differentiating it correctly. There was a penalty for failing to differentiate the constant correctly but this was rarely applied.

9) The vast majority scored full marks here. There were various initial moves, the commonest was $3x\log 5 = \log 100$. A neat move seen a few times was $x\log 125 = \log 100$. Those who started $3x=\log_5 100$ seemed to be inviting trouble but their calculators seemed to cope with base 5.

Section B

10) (i) In spite of hard evidence and some strong clues against it, angle SLT was taken to be 105° by perhaps half the entry. As 105 does appear in the space between the arms of that angle, this misinterpretation was not heavily punished. There were two method marks for applying the cosine rule and one for the sine rule, whatever number they took for angle SLT. The bearing mark was allowed if it correctly followed from the candidate's value for angle LTS or angle LST. Many failed to calculate the required bearing.

(ii) Some had difficulty finding the distance travelled, 24x26/60=10.4, especially if they converted 26 minutes to 0.43 hours as a first step. The formula $s=r\theta$ was well known, just a few used $s=r^2\theta/2$. Most earned the mark for converting their 2 radians to degrees. Many candidates could not resist the temptation to see the diagrams as being symmetrical about the N/S line. Again not many successfully found the required bearing.

11) (i) Most knew exactly what they had to do here to earn the five marks and those who took care not to make silly mistakes did earn them. Some made a slip with the derivative, $2x^2-6x$ perhaps. Some factorised $3x^2-6x$ as $3x\,(x-3)$ or worse. Weaker candidates did not use $y'=0$ to find the stationary points, some tried $y=0$ or $y''=0$. The commonest test used was 'for a maximum, y'' is negative'.

(ii) Many candidates produced excellent work here. Candidates essentially had to solve $y'=9$, pick the root $x=3$ for point P, find $y(3)$, find the gradient of the normal and then the equation of the normal. That was quite a series of operations and many were successful. The work came off the rails when they thought that P was the point where the tangent at (-1, -3) met the curve again. Candidates earned a mark for finding the two intercepts made by their normal. For the final mark their normal and intercepts had to be correct forming a triangle of area 8. A few took a rather longer route using definite integration for the normal from 0 to 12.

12) (i) There was a full range of responses here; everyone knew they had to log both sides but some were more successful than others. The commonest error was loga x bt log10. Many did not convert bt log10 to bt.

(ii) Most could see that 2 significant figures were insufficient so they gave 3 but there was much carelessness with the rounding; 3.897 became 3.89 and 4.107 became 4.10 or 4.12. They received a mark for plotting their values with a 1mm tolerance and they received a mark for a reasonable and ruled line of best fit, using their points.

(iii) Many candidates found the gradient of their line and wrote down their intercept but were not sure whether to use them in a formula for P or for log P. Two marks were awarded if they were seen, two more if they were correctly used.

(iv) As many candidates arrived at a prediction by extending their graph and axes these two marks were awarded for any answer in a reasonable range; a single mark was awarded for a wider range. There were many excellent attempts scoring full marks, weaker candidates just scored on the graph or not at all.