RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4752/01**

MATHEMATICS (MEI)

Concepts for Advanced Mathematics (C2)

THURSDAY 15 MAY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Insert for Question 13
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- There is an **insert** for use in Question **13**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **7** printed pages, **1** blank page and an insert.

Section A (36 marks)

1 Express $\frac{7\pi}{6}$ radians in degrees. [2]

2 The first term of a geometric series is 5.4 and the common ratio is 0.1.

(i) Find the fourth term of the series. [1]

(ii) Find the sum to infinity of the series. [2]

3 State the transformation which maps the graph of $y = x^2 + 5$ onto the graph of $y = 3x^2 + 15$. [2]

4 Use calculus to find the set of values of x for which $f(x) = 12x - x^3$ is an increasing function. [3]

5 In Fig. 5, A and B are the points on the curve $y = 2^x$ with x-coordinates 3 and 3.1 respectively.

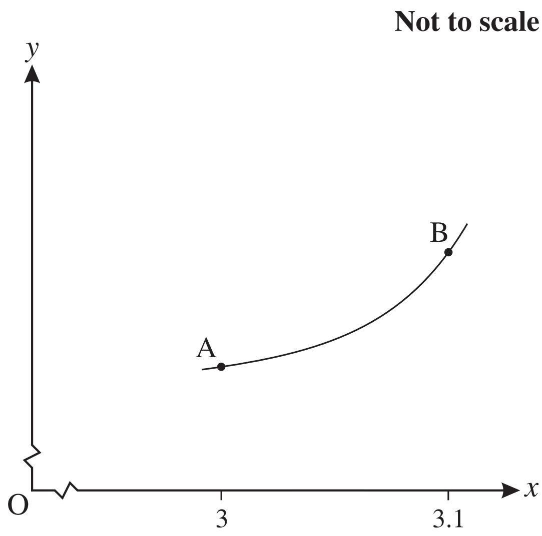

Fig. 5

(i) Find the gradient of the chord AB. Give your answer correct to 2 decimal places. [2]

(ii) Stating the points you use, find the gradient of another chord which will give a closer approximation to the gradient of the tangent to $y = 2^x$ at A. [2]

6 A curve has gradient given by $\frac{dy}{dx} = 6\sqrt{x}$. Find the equation of the curve, given that it passes through the point (9, 105). [4]

7

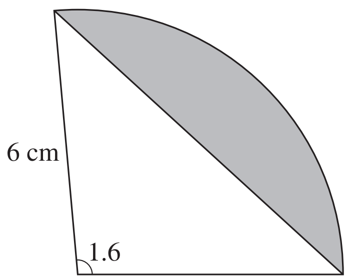

Fig. 7

A sector of a circle of radius 6 cm has angle 1.6 radians, as shown in Fig. 7.

Find the area of the sector.

Hence find the area of the shaded segment. [5]

8 The 11th term of an arithmetic progression is 1. The sum of the first 10 terms is 120. Find the 4th term. [5]

9 Use logarithms to solve the equation $5^x = 235$, giving your answer correct to 2 decimal places. [3]

10 Showing your method, solve the equation $2\sin^2\theta = \cos\theta + 2$ for values of θ between $0°$ and $360°$. [5]

Section B (36 marks)

11

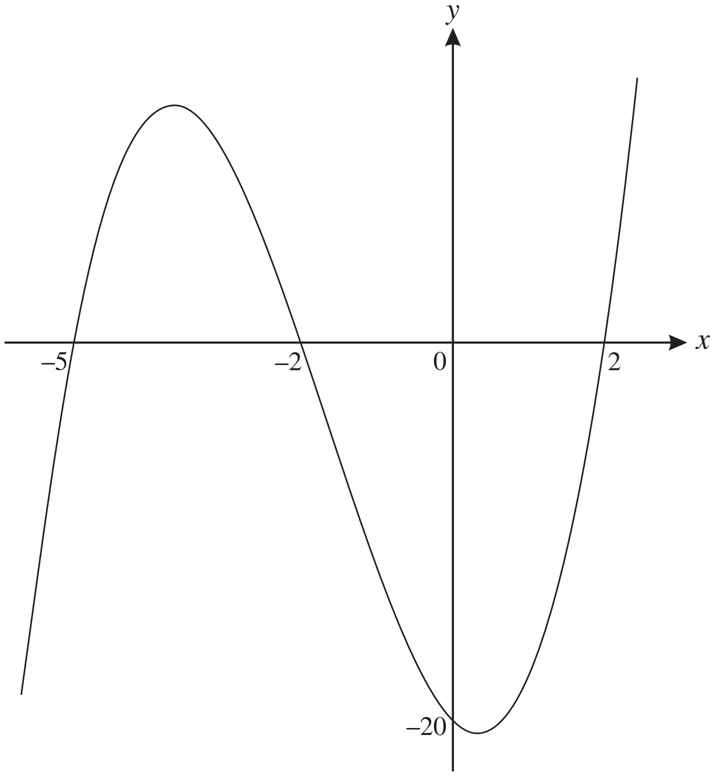

Fig. 11

Fig. 11 shows a sketch of the cubic curve $y = \mathrm{f}(x)$. The values of x where it crosses the x-axis are -5, -2 and 2, and it crosses the y-axis at $(0, -20)$.

(i) Express $\mathrm{f}(x)$ in factorised form. [2]

(ii) Show that the equation of the curve may be written as $y = x^3 + 5x^2 - 4x - 20$. [2]

(iii) Use calculus to show that, correct to 1 decimal place, the x-coordinate of the minimum point on the curve is 0.4.

Find also the coordinates of the maximum point on the curve, giving your answers correct to 1 decimal place. [6]

(iv) State, correct to 1 decimal place, the coordinates of the maximum point on the curve $y = \mathrm{f}(2x)$. [2]

12

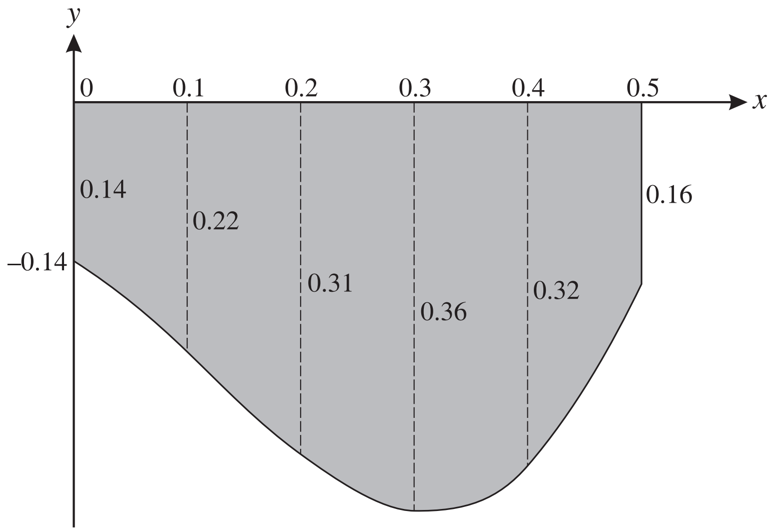

Fig. 12

A water trough is a prism 2.5 m long. Fig. 12 shows the cross-section of the trough, with the depths in metres at 0.1 m intervals across the trough. The trough is full of water.

(i) Use the trapezium rule with 5 strips to calculate an estimate of the area of cross-section of the trough.

Hence estimate the volume of water in the trough. [5]

(ii) A computer program models the curve of the base of the trough, with axes as shown and units in metres, using the equation $y = 8x^3 - 3x^2 - 0.5x - 0.15$, for $0 \leqslant x \leqslant 0.5$.

Calculate $\int_0^{0.5} (8x^3 - 3x^2 - 0.5x - 0.15)\,dx$ and state what this represents.

Hence find the volume of water in the trough as given by this model. [7]

[Question 13 is printed overleaf.]

13 The percentage of the adult population visiting the cinema in Great Britain has tended to increase since the 1980s. The table shows the results of surveys in various years.

Year	1986/87	1991/92	1996/97	1999/00	2000/01	2001/02
Percentage of the adult population visiting the cinema	31	44	54	56	55	57

Source: Department of National Statistics, www.statistics.gov.uk

This growth may be modelled by an equation of the form

$$P = at^b,$$

where P is the percentage of the adult population visiting the cinema, t is the number of years after the year 1985/86 and a and b are constants to be determined.

(i) Show that, according to this model, the graph of $\log_{10} P$ against $\log_{10} t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis. [3]

Answer part (ii) of this question on the insert provided.

(ii) Complete the table of values on the insert, and plot $\log_{10} P$ against $\log_{10} t$. Draw by eye a line of best fit for the data. [4]

(iii) Use your graph to find the equation for P in terms of t. [4]

(iv) Predict the percentage of the adult population visiting the cinema in the year 2007/2008 (i.e. when $t = 22$), according to this model. [1]

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Acknowledgements:

Q.13 Source: Department of National Statistics, www.statistics.gov.uk. Crown copyright material is reproduced with the permission of the Controller of HMSO and the Queen's Printer for Scotland.

OCR RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4752/01**

MATHEMATICS (MEI)

Concepts for Advanced Mathematics (C2)

INSERT for Question 13

THURSDAY 15 MAY 2008 Morning

Time: 1 hour 30 minutes

Candidate Forename		Candidate Surname	
Centre Number		Candidate Number	

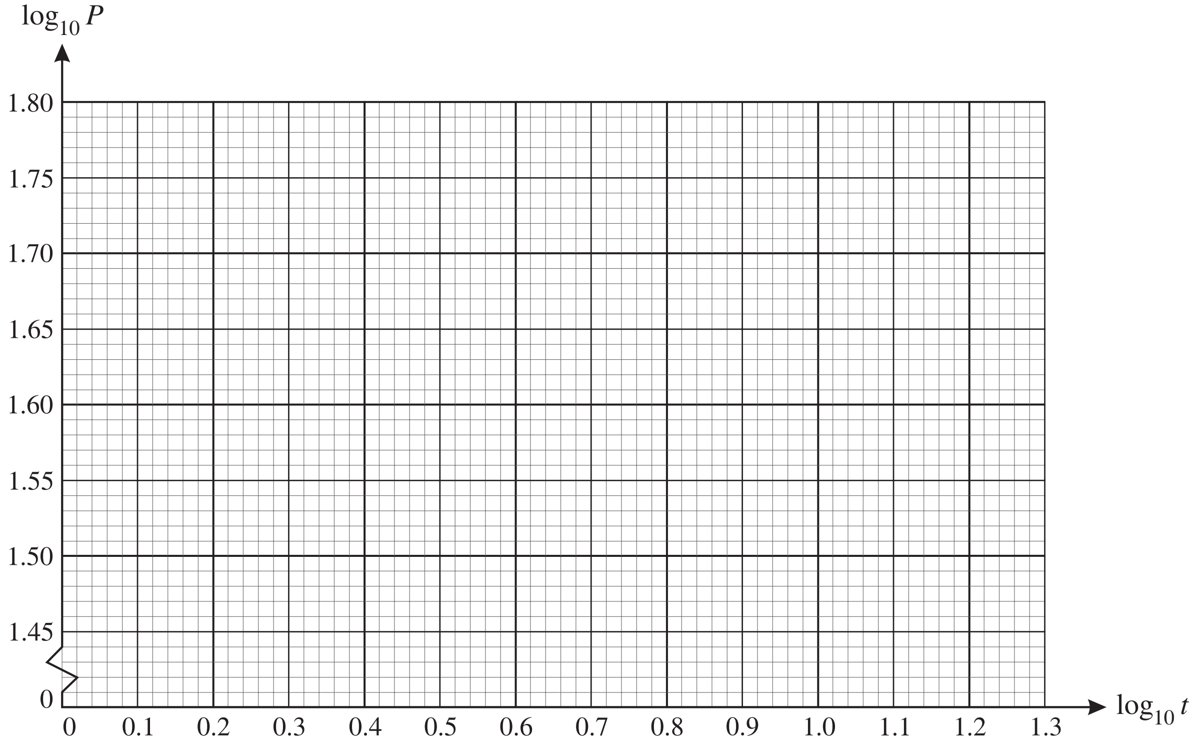
INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- This insert should be used to answer Question **13 (ii)**.
- Write your answers to Question **13 (ii)** in the spaces provided in this insert, and **attach it to your answer booklet**.

This document consists of **2** printed pages.

13 (ii)

Year	1986/87	1991/92	1996/97	1999/00	2000/01	2001/02
t	1	6	11	14	15	16
P	31	44	54	56	55	57
$\log_{10} t$			1.04			
$\log_{10} P$			1.73			



4752 (C2) Concepts for Advanced Mathematics

Section A

1	210 c.a.o.	2	1 for π rads = 180° soi	2
2	(i) 5.4×10^{-3}, 0.0054 or $\frac{27}{5000}$ (ii) 6 www	1 2	 M1 for S = 5.4 / (1 − 0.1)	 3
3	stretch, parallel to the y axis, sf 3	2	1 for stretch plus one other element correct	2
4	[$f'(x)$ =] $12 - 3x^2$ their $f'(x) > 0$ or = 0 soi $-2 < x < 2$	B1 M1 A1	 condone $-2 \le x \le 2$ or "between -2 and 2"	 3
5	(i) grad of chord = $(2^{3.1} - 2^3)/0.1$ o.e. $= 5.74$ c.a.o. (ii) correct use of A and C where for C, $2.9 < x < 3.1$ answer in range $(5.36, 5.74)$	M1 A1 M1 A1	 or chord with ends x = 3 ± h, where 0 < h ≤ 0.1 s.c.1 for consistent use of reciprocal of gradient formula in parts (i) and (ii)	 4
6	[y =] $kx^{3/2}$ [+ c] k = 4 subst of (9, 105) in their eqn with c or $c = -3$	M1 A1 M1 A1	 may appear at any stage must have c; must have attempted integration	 4
7	sector area = 28.8 or $\frac{144}{5}$ [cm²] c.a.o. area of triangle = $\frac{1}{2} \times 6^2 \times \sin 1.6$ o.e. their sector – their triangle s.o.i. 10.8 to 10.81 [cm²]	2 M1 M1 A1	M1 for $\frac{1}{2} \times 6^2 \times 1.6$ must both be areas leading to a positive answer	 5
8	$a + 10d = 1$ *or* 121 = 5.5(2a+10d) $5(2a + 9d) = 120$ o.e. $a = 21$ s.o.i. www and $d = -2$ s.o.i. www 4th term is 15	M1 M1 A1 A1 A1	or 121 = 5.5(a + 1) gets M2 eg $2a + 9d = 24$	 5
9	$x \log 5 = \log 235$ or $x = \frac{\log 235}{\log 5}$ 3.39	M1 A2	or $x = \log_5 235$ A1 for 3.4 or versions of 3.392…	 3
10	$2(1 - \cos^2\theta) = \cos\theta + 2$ $-2\cos^2\theta = \cos\theta$ s.o.i. valid attempt at solving their quadratic in $\cos\theta$ $\cos\theta = -\frac{1}{2}$ www θ = 90, 270, 120, 240	M1 A1 DM1 A1 A1	for $1 - \cos^2\theta = \sin^2\theta$ substituted graphic calc method: allow M3 for intersection of $y = 2\sin^2\theta$ and $y = \cos\theta + 2$ and A2 for all four roots. All four answers correct but unsupported scores B2. 120 and 240 only: B1.	 5

18

18

Section B

11	**i**	$(x + 5)(x - 2)(x + 2)$	2	M1 for $a\ (x + 5)(x - 2)(x + 2)$	2
	ii	$[(x + 2)](x^2 + 3x - 10)$ $x^3 + 3x^2 - 10x + 2x^2 + 6x - 20$ o.e.	M1 M1	for correct expansion of one pair of their brackets for clear expansion of correct factors – accept given answer from $(x + 5)(x^2 - 4)$ as first step	 2
	iii	$y' = 3x^2 + 10x - 4$ their $3x^2 + 10x - 4 = 0$ s.o.i. x = 0.36… from formula o.e. (−3.7, 12.6)	M2 M1 A1 B1+1	M1 if one error or M1 for substitution of 0.4 if trying to obtain 0, and A1 for correct demonstration of sign change	 6
	iv	(-1.8, 12.6)	B1+1	accept (−1.9, 12.6) or f.t.(½ their max x, their max y)	2
12	**i**	Area = (-)0.136 seen [m^2] www Volume = 0.34 [m^3] or ft from their area × 2.5	4 1	M3 for 0.1/2 × (0.14 + 0.16 + 2[0.22 + 0.31 + 0.36 + 0.32]) M2 for one slip; M1 for two slips must be positive	 5
	ii	$2x^4 - x^3 - 0.25\ x^2 - 0.15x$ o.e. value at 0.5 [– value at 0] = –0.1375 area of cross section (of trough) or area between curve and x-axis 0.34375 r.o.t. to 3 or more sf [m^3] m^3 seen in (i) or (ii)	M2 M1 A1 E1 B1 U1	M1 for 2 terms correct dep on integral attempted must have neg sign	 7
13	**i**	$\log P = \log a + b \log t$ www comparison with $y = mx + c$ intercept = $\log_{10} a$	1 1 1	 must be with correct equation condone omission of base	 3
	ii	log t 0 0.78 1.15 1.18 1.20 log P 1.49 1.64 1.75 1.74 1.76 plots f.t. ruled line of best fit	1 1 1 1	accept to 2 or more dp	 4
	iii	gradient rounding to 0.22 or 0.23 $a = 10^{1.49}$ s.o.i. $P = 31t^{\ m}$ allow the form $P = 10^{0.22\log t + 1.49}$	2 1 1	M1 for y step / x-step accept1.47 – 1.50 for intercept accept answers that round to 30 – 32 , their positive m	 4
	iv	answer rounds in range 60 to 63	1		1

4752 Concepts for Advanced Mathematics (C2)

General Comments

The paper was accessible to nearly all candidates, but there was enough to challenge even the best. There was no evidence that candidates struggled for time. Some candidates set their work out fully and clearly. Nevertheless, many marks were lost by failing to show sufficient detail of the method, using a method different to the one specified in the question or, in a small number of cases, by contriving a new question and answering that instead. Some also lost marks by failing to leave answers in their simplest form, or by rounding prematurely.

Comments on Individual Questions

Section A

1) This question was generally very well done. A few candidates simply converted $\frac{7}{6}\pi$ to a decimal, and some were unsure what to do with "π radians = 180°".

2) This question was very well done, with the majority scoring full marks. Some candidates evaluated S_4 in part (i), and a few decided the series was arithmetic and gave the fourth term as 5.7. Even so, they often went on to score both marks in part (ii).

3) This was done very badly, even by very strong candidates. Many correctly obtained y = 3f(*x*), but did not make any further progress. Those who did describe the transformation correctly often spoiled their answer by adding a second one – usually a vertical translation of +10.

4) Most candidates obtained the first two marks, but only a few went on to give the correct answer. Common errors were "$x > 2$ or $x < -2$" and "$-2 > x < 2$". Some weaker candidates ignored the instruction to use calculus, and simply evaluated the function for various values of *x*, which usually led to the wrong conclusion (and scored zero anyway). A few others thought that "increasing function" meant "f(*x*) > 0" and did quite a bit of work but failed to score.

5) Most did well on this question. Problems in part (i) usually arose from premature rounding. A minority obtained 5.7 and then changed this to 5.70 in response to the instruction to give the answer to two decimal places. Similarly, in part (ii), a correct method was often spoiled by premature rounding leading to a *less* accurate answer. A few candidates used new *x*-values such as 3.02 and 3.05, which didn't score, and a few others simply used the midpoint of the original chord, obtaining the same answer as in part (i). A handful completely missed the point, and wrote answers such as $x2^{x-1}$.

6) This was often done very well. A few encountered difficulty in evaluating 6 / 1.5, or simply left it unsimplified. A significant minority thought $6\sqrt{x}$ was *x* to the power $\frac{1}{6}$, and then integrated. Another minority took the gradient to be "$6\sqrt{x}$" and then substituted in y = m*x* + c.

7) Most used $\frac{1}{2}r^2\theta$ correctly, but a few candidates converted θ to degrees and then used the formula. A handful used the formula $\frac{1}{2}r\theta^2$ and scored zero. In the second part, almost everyone knew what was required, but some candidates wrongly assumed the triangle was right angled, and used ½ base × height. Some candidates wasted much time by using the cosine rule to find the "base" and then dropping a perpendicular for the "height", and occasionally penalised themselves even more by rounding prematurely.

8) Many candidates were able to write down two correct equations, but a significant minority then made errors with the algebra when eliminating a or d. Weaker candidates wrote "11 = a + 10d" and "120 = 5(2a + 119d)" was seen occasionally. Some of the better candidates went straight to 121 = 5.5(a + 1) and reached the correct solution quickly. Some weaker candidates could not adopt an algebraic approach, but contrived the solution correctly by trial and error.

9) Nearly all candidates scored full marks on this question. It seemed that many had the facility on their calculator to evaluate $\log_5 235$ directly. No marks were given for an unsupported answer. Some carelessly rounded to 3.4 and lost a mark; a few gave the answer as 3.40 and lost both marks. Weaker candidates ignored the instruction to use logarithms, and used trial and error, which scored zero.

10) This was generally not well done, with only a minority scoring full marks. Many knew to use Pythagoras, but made errors in substitution, such as "$\cos\theta = 1 - \sin\theta$", "$\sin^2\theta = \cos^2\theta - 1$", "$2 - \cos^2\theta = \cos\theta + 2$" or even "$\sin^2\theta = 1/\cos^2\theta$". Those that did manage to produce the correct quadratic often made errors in factorising to obtain $\cos\theta = \frac{1}{2}$.

Section B

11) (i) This was generally very well done.

(ii) Most candidates correctly expanded the brackets from part (i), but some simply evaluated f(-5), f(-2) and f(2), which didn't score.

(iii) Most candidates attempted to solve $\frac{dy}{dx} = 3x^2 + 10x - 4 = 0$, but many spoiled their solutions by simply writing down the answer as 0.4. Candidates need to be reminded that full working is expected for answers which are given. A significant minority of candidates wasted time by considering the second derivative to demonstrate the nature of the turning points. This was not asked for, and gained no credit. A good number failed to give the co-ordinates of the maximum point to the required accuracy.

(iv) Many failed to use the answer to part (iii) and wasted much time re-working the equation – frequently making errors in the process. Those who did use the answers from part (iii) often doubled *x* or *y* or both.

12) (i) This was often very well done, with many candidates scoring full marks. However, those who were unable to quote the formula correctly were heavily penalised, as it is provided in the formula booklet. Common errors were the omission of a pair of brackets, which was often irrevocable, and "h = 2.5" or " h = 0.5". Most knew to multiply their answer by 2.5 to obtain the volume, but a few multiplied by 100. Most gave the correct units.

(ii) The integral was often correctly done. Where slips were made, by far the most common was to give the last term as "$0.15^2/2$". A few candidates found $\frac{dy}{dx}$ and substituted in 0.5, and a few others simply found f(0.5). Many candidates were not able to say what the integral represented, and made comments about over-estimates and under-estimates.

13) (i) This was often well done, but those who tried to derive the result often made the error "$\log at^b = b \log at$", and a significant minority failed to state the intercept.

(ii) The table of results was largely correct, with just the odd slip, and most were able to plot the points correctly. A common mistake was to plot (0.078, 1.64) instead of (0.78, 1.64). Lines of best fit were generally ruled and sensible.

(iii) Most knew how to find the gradient of their line, but frequently made arithmetical errors – usually by being out by a factor of 10 in the numerator. "a" was often found correctly, but only the better candidates were able to produce a correct version of the equation. Marks were often lost at this stage for inaccurate work on the insert leading to answers outside the acceptable range.

(iv) Those who had done well in the earlier parts had no difficulty here. In addition, many candidates scored zero in parts (i) and (iii), yet still managed to obtain this mark by extrapolating from the table of values.