1) $\frac{x}{x^2-1} + \frac{2}{x+1}$

$= \frac{x}{(x+1)(x-1)} + \frac{2}{x+1}$

$= \frac{x + 2(x-1)}{(x+1)(x-1)}$

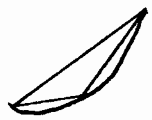
$= \frac{3x-2}{(x+1)(x-1)}$

2) $x = 0.5 \Rightarrow y = 1.1180$

i) $A = \frac{h}{2}\left[y_0 + 2(y_1+y_2+y_3)+y_4\right]$

$A = \frac{0.25}{2}\left[1 + 2(1.0308 + 1.1180 + 1.25) + 1.4142\right]$

$= 1.151475 \approx 1.151 \text{ units}^2$

ii) 

For this concave curve the trapezium rule overestimates the area as the trapeziums extend above the curve.

Increasing the number of strips reduces the error so Jenny should have obtained an answer less than 1.151

iii) $V = \pi \int_0^1 y^2 \, dx$

$V = \pi \int (1+x^2) \, dx$

$V = \pi \left[x + \frac{x^3}{3}\right]_0^1$

$V = \pi\left[\left(1+\frac{1}{3}\right) - (0+0)\right]$

$V = \frac{4\pi}{3} \text{ units}^3$

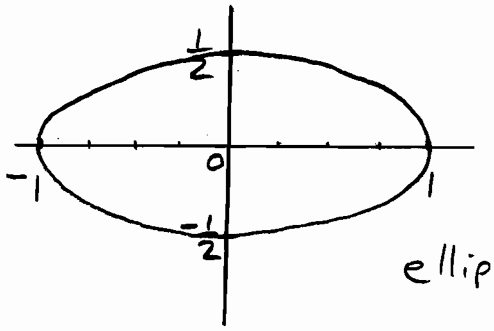
3) $x = \cos 2\theta, \quad y = \sin\theta\cos\theta$

$\Rightarrow x = \cos 2\theta, \quad y = \frac{1}{2}\sin 2\theta$

$\Rightarrow x = \cos 2\theta, \quad 2y = \sin 2\theta$

$\Rightarrow x^2 + (2y)^2 = \cos^2 2\theta + \sin^2 2\theta$

$\Rightarrow x^2 + 4y^2 = 1$



ellipse

4) $\sqrt{4+x} = \sqrt{4\left(1+\frac{x}{4}\right)}$

$= 2\sqrt{1+\frac{x}{4}}$

$= 2\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$

$\approx 2\left[1 + \frac{1}{2}\left(\frac{x}{4}\right) + \frac{\frac{1}{2}\cdot -\frac{1}{2}}{1\cdot 2}\left(\frac{x}{4}\right)^2\right]$

4 cont) $= 2\left[1 + \frac{x}{8} - \frac{x^2}{128}\right]$

$= 2 + \frac{x}{4} - \frac{x^2}{64}$

Valid for $\left|\frac{x}{4}\right| < 1$

$-4 < x < 4$

5)

i) $\frac{3}{(y-2)(y+1)} \equiv \frac{A}{y-2} + \frac{B}{y+1}$

$3 \equiv A(y+1) + B(y-2)$

Put $y = -1$

$3 = B(-1-2)$
$3 = -3B \Rightarrow B = -1$

Put $y = 2$

$3 = A(2+1)$
$3 = 3A \Rightarrow A = 1$

$\therefore \frac{3}{(y-2)(y+1)} \equiv \frac{1}{y-2} - \frac{1}{y+1}$

ii) $\frac{dy}{dx} = x^2(y-2)(y+1)$

$\Rightarrow \int \frac{1}{(y-2)(y+1)}\,dy = \int x^2\,dx$

$\Rightarrow \int \frac{3}{(y-2)(y+1)}\,dy = \int 3x^2\,dx$

$\Rightarrow \int \left(\frac{1}{y-2} - \frac{1}{y-1}\right) dy = \int 3x^2\,dx$

$\Rightarrow \ln(y-2) - \ln(y+1) = x^3 + c$

$\Rightarrow \ln\left(\frac{y-2}{y+1}\right) = x^3 + c$

$\Rightarrow \left(\frac{y-2}{y+1}\right) = e^{x^3+c}$

$\Rightarrow \left(\frac{y-2}{y+1}\right) = e^{x^3} \times e^{c}$

$\Rightarrow \left(\frac{y-2}{y+1}\right) = Ae^{x^3}$

where $A = e^c$

6) $\tan(\theta + 45°) = 1 - 2\tan\theta$

$\frac{\tan\theta + \tan 45°}{1 - \tan\theta \tan 45°} = 1 - 2\tan\theta$

$\frac{\tan\theta + 1}{1 - \tan\theta} = 1 - 2\tan\theta$

$\tan\theta + 1 = (1 - 2\tan\theta)(1 - \tan\theta)$

$\tan\theta + 1 = 1 - 2\tan\theta - \tan\theta + 2\tan^2\theta$

$0 = 2\tan^2\theta - 4\tan\theta$

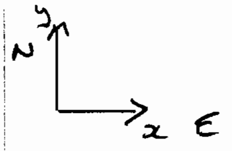
$0 = 2\tan\theta(\tan\theta - 2)$

$\Rightarrow \tan\theta = 0$ or $\tan\theta = 2$

$\Rightarrow \theta = 0$ or $\theta = 63.43°$

for $0 \le \theta \le 90°$

7) i)



$A(-200, 100, 0)$
$B(100, 200, 100)$

$$\vec{AB} = \begin{pmatrix} 100 - -200 \\ 200 - 100 \\ 100 - 0 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$$

$$|\vec{AB}| = \sqrt{300^2 + 100^2 + 100^2}$$

$$|\vec{AB}| = 331.66 \text{ m}$$

ii) $$\underline{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$$

Vertical vector $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

If angle with vertical is θ

$$\cos\theta = \frac{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}}{\left|\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right| \left|\begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}\right|}$$

$$\cos\theta = \frac{0 + 0 + 100}{1 \times 331.66}$$

$$\theta = 72.45^\circ$$

iii) Plane is $x + 2y + 3z = 320$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -200 + 300\lambda \\ 100 + 100\lambda \\ 100\lambda \end{pmatrix}$$

Subst in plane

$$-200 + 300\lambda + 2(100 + 100\lambda) + 3(100\lambda) = 320$$

$$\cancel{-200} + 300\lambda + \cancel{200} + 200\lambda + 300\lambda = 320$$

$$800\lambda = 320$$

$$\lambda = \frac{320}{800} = 0.4$$

Subst for λ in line

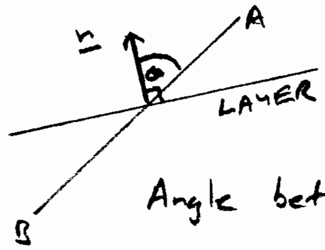
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -200 + 300 \times 0.4 \\ 100 + 100 \times 0.4 \\ 100 \times 0.4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -80 \\ 140 \\ 40 \end{pmatrix}$$

Pipeline meets rock at

$$(-80, 140, 40)$$

iv)



$$\text{normal} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Angle between AB and normal

$$\cos\theta = \frac{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} . \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\right| \left|\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}\right|}$$

$$\cos\theta = \frac{3 + 2 + 3}{\sqrt{14}\sqrt{11}} = \frac{8}{\sqrt{154}}$$

$$\theta = 49.86^\circ$$

Cuts thro layer at $90 - \theta = 40.14^\circ$

8) $x = 2\theta - \sin\theta, \quad y = 4\cos\theta$

i) At A $y = 0 \Rightarrow \cos\theta = 0$

$$\Rightarrow \theta = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}$$

when $\theta = \frac{\pi}{2}$, $x = \pi - 1$

when $\theta = \frac{3\pi}{2}$, $x = 3\pi + 1$

so A is point $(\pi - 1, 0)$

with parameter $\theta = \frac{\pi}{2}$

At B y has minimum value

$$\Rightarrow \cos\theta = -1$$

$$\Rightarrow \theta = \pi$$

When $\theta = \pi$, $x = 2\pi - \sin\pi$
$x = 2\pi$
$y = 4\cos\pi = -4$

so B is point $(2\pi, -4)$

$OA = \pi - 1$

$$\begin{aligned} AC &= OC - OA \\ &= 2\pi - (\pi - 1) \\ &= \pi + 1 \end{aligned}$$

$\therefore$ OA : AC
$= (\pi - 1) : (\pi + 1)$

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ii) $\frac{dy}{d\theta} = -4\sin\theta$

$\frac{dx}{d\theta} = 2 - \cos\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{-4\sin\theta}{2 - \cos\theta}$$

At A, with $\theta = \frac{\pi}{2}$

$$\frac{dy}{dx} = \frac{-4\sin\frac{\pi}{2}}{2 - \cos\frac{\pi}{2}} = \frac{-4}{2-0}$$

$\therefore$ gradient at A is -2

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iii) If $\frac{dy}{dx} = 1$ then $\frac{-4\sin\theta}{2 - \cos\theta} = 1$

$$\Rightarrow -4\sin\theta = 2 - \cos\theta$$

$$\Rightarrow \cos\theta - 4\sin\theta = 2$$

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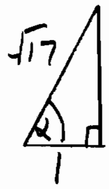
iv) $\cos\theta - 4\sin\theta \qquad (\cos\theta\cos\alpha - \sin\theta\sin\alpha)$



$$= \sqrt{17}\left(\frac{1}{\sqrt{17}}\cos\theta - \frac{4}{\sqrt{17}}\sin\theta\right)$$

$$= \sqrt{17}\cos(\theta + \alpha)$$

where $\alpha = \tan^{-1}4 = 1.326$ rads

$$\sqrt{17}\cos(\theta + 1.326) = 2$$

$$\cos(\theta + 1.326) = \frac{2}{\sqrt{17}}$$

$\theta + 1.326 = 1.064, \ 2\pi - 1.064, \ 2\pi + 1.064$

$\theta = \cancel{1.064 - 1.326},$

$\theta = 2\pi - 1.064 - 1.326 = 3.893$ rads

$\theta = 2\pi + 1.064 - 1.326 = 6.021$ rads