OCR
RECOGNISING ACHIEVEMENT

Thursday 13 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 **(i)** Express $\dfrac{x}{(1+x)(1-2x)}$ in partial fractions. [3]

(ii) Hence use binomial expansions to show that $\dfrac{x}{(1+x)(1-2x)} = ax + bx^2 + \ldots$, where a and b are constants to be determined.

State the set of values of x for which the expansion is valid. [5]

2 Show that the equation $\operatorname{cosec} x + 5\cot x = 3\sin x$ may be rearranged as

$$3\cos^2 x + 5\cos x - 2 = 0.$$

Hence solve the equation for $0° \leqslant x \leqslant 360°$, giving your answers to 1 decimal place. [7]

3 Using appropriate right-angled triangles, show that $\tan 45° = 1$ and $\tan 30° = \dfrac{1}{\sqrt{3}}$.

Hence show that $\tan 75° = 2 + \sqrt{3}$. [7]

4 **(i)** Find a vector equation of the line l joining the points $(0, 1, 3)$ and $(-2, 2, 5)$. [2]

(ii) Find the point of intersection of the line l with the plane $x + 3y + 2z = 4$. [3]

(iii) Find the acute angle between the line l and the normal to the plane. [3]

5 The points A, B and C have coordinates $\mathrm{A}(3, 2, -1)$, $\mathrm{B}(-1, 1, 2)$ and $\mathrm{C}(10, 5, -5)$, relative to the origin O. Show that $\overrightarrow{\mathrm{OC}}$ can be written in the form $\lambda\overrightarrow{\mathrm{OA}} + \mu\overrightarrow{\mathrm{OB}}$, where λ and μ are to be determined.

What can you deduce about the points O, A, B and C from the fact that $\overrightarrow{\mathrm{OC}}$ can be expressed as a combination of $\overrightarrow{\mathrm{OA}}$ and $\overrightarrow{\mathrm{OB}}$? [6]

Section B (36 marks)

6 The motion of a particle is modelled by the differential equation

$$v\frac{\mathrm{d}v}{\mathrm{d}x} + 4x = 0,$$

where x is its displacement from a fixed point, and v is its velocity.

Initially $x = 1$ and $v = 4$.

(i) Solve the differential equation to show that $v^2 = 20 - 4x^2$. **[4]**

Now consider motion for which $x = \cos 2t + 2\sin 2t$, where x is the displacement from a fixed point at time t.

(ii) Verify that, when $t = 0$, $x = 1$. Use the fact that $v = \frac{\mathrm{d}x}{\mathrm{d}t}$ to verify that when $t = 0$, $v = 4$. **[4]**

(iii) Express x in the form $R\cos(2t - \alpha)$, where R and α are constants to be determined, and obtain the corresponding expression for v. Hence or otherwise verify that, for this motion too, $v^2 = 20 - 4x^2$. **[7]**

(iv) Use your answers to part **(iii)** to find the maximum value of x, and the earliest time at which x reaches this maximum value. **[3]**

7 Fig. 7 shows the curve BC defined by the parametric equations

$$x = 5\ln u,\ y = u + \frac{1}{u},\quad 1 \leqslant u \leqslant 10.$$

The point A lies on the x-axis and AC is parallel to the y-axis. The tangent to the curve at C makes an angle θ with AC, as shown.

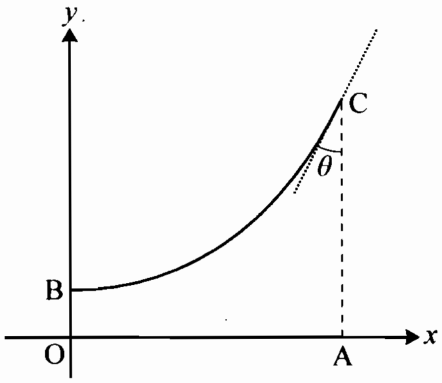

Fig. 7

(i) Find the lengths OA, OB and AC. **[5]**

(ii) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of u. Hence find the angle θ. **[6]**

(iii) Show that the cartesian equation of the curve is $y = \mathrm{e}^{\frac{1}{5}x} + \mathrm{e}^{-\frac{1}{5}x}$. **[2]**

An object is formed by rotating the region OACB through 360° about Ox.

(iv) Find the volume of the object. **[5]**