1) i) n an integer

P: n is an even number
Q: n is a multiple of 4

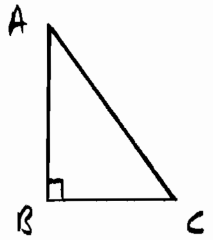
A multiple of 4 is even so $P \Leftarrow Q$

However, 2 is even but not a multiple of 4 so $P \not\Rightarrow Q$

Answer $P \Leftarrow Q$

ii) For $\triangle ABC$

P: $B = 90^\circ$
Q: $AB^2 + BC^2 = AC^2$



Pythagoras Theorem and its converse

$P \Leftrightarrow Q$

2) n, $n+1$, $n+2$

Let any 3 consecutive integers be n, $n+1$, $n+2$

Then
$n + n+1 + n+2$

$= 3n+3 = 3(n+1)$

3 is a factor so sum is divisible by 3

3) n is a positive integer

$n^2 + n = n(n+1)$

If n is odd $n+1$ is even

If n is even $n+1$ is odd

In both cases $n(n+1)$ has an even factor and so is even

4) i) P: $x^2 + x - 2 = 0$

Q: $x = 1$

If $x = 1$, $1^2 + 1 - 2 = 0$

$\therefore$ $P \Leftarrow Q$

However, $x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

so $x = 1$ or $x = -2$

$\therefore$ $P \not\Rightarrow Q$

Answer $P \Leftarrow Q$

ii) P: $y^3 > 1$

Q: $y > 1$

$P \Leftrightarrow Q$

5) n an odd integer $\Rightarrow 2n$ an even integer

Converse:

$2n$ an even integer $\Rightarrow n$ an odd integer

Not true

2×4 an even integer

but 4 is an even integer

6) i) If n is an even integer

let $n = 2m$ where m an integer

$\therefore \; 3n^2 + 6n$

$= 3(2m)^2 + 6(2m)$

$= 3 \times 4m^2 + 12m$

$= 12m^2 + 12m$

$= 12(m^2 + m)$

$\therefore$ divisible by 12

since $m^2 + m$ is an integer

6 ii) Let $n = 1$

$3n^2 + 6n = 3 + 6 = 9$

so 12 is not a factor of

$3n^2 + 6n$ for all +ve integers n

7) n is a positive integer

i) $2n + 1$ is odd integer TRUE

ii) $3n+1$ is even integer EITHER

Since $3(2)+1 = 7$
$3(3)+1 = 10$

iii) n is odd $\Rightarrow n^2$ odd TRUE

iv) n^2 is odd $\Rightarrow n^3$ is even FALSE

since n^2 odd
$\Rightarrow n$ odd
$\Rightarrow n^3$ odd

8) n an integer

Consider $n^3 - n$

If n is odd $\quad n^3 = \text{odd} \times \text{odd} \times \text{odd}$
$= \text{odd}$

$n^3 - n = \text{odd} - \text{odd} = \text{even}$

If n is even

$n^3 = \text{even} \times \text{even} \times \text{even}$
$= \text{even}$

$n^3 - n = \text{even} - \text{even} = \text{even}$

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