MEI CORE 1 QUADRATIC FUNCTIONS REVISION

1) i) $y = x^2 - 3x + 11$

$y = (x - \frac{3}{2})^2 + 11 - \frac{9}{4}$

$y = (x - \frac{3}{2})^2 + \frac{35}{4}$

This graph has a minimum point at $(\frac{3}{2}, \frac{35}{4})$ and so is always above x-axis

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ii) $y = 2x^2 + x - 10$

Above x-axis when

$2x^2 + x - 10 > 0$

$2x - 10 = -20$

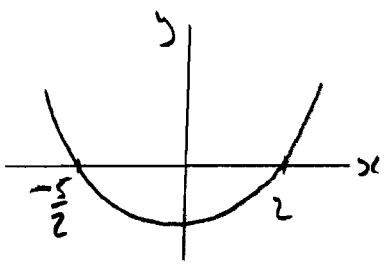
Factors of -20 which add to $+1$ are $+5$ and -4

$2x^2 + 5x - 4x - 10 > 0$

$x(2x+5) - 2(2x+5) > 0$

$(x-2)(2x+5) > 0$

Sketch $y = (x-2)(2x+5)$



Solution $x > 2$

or $x < -\frac{5}{2}$

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iii) $y = x^2 - 3x + 11$ ①

$y = 2x^2 + x - 10$ ②

Sub for y in ①

$2x^2 + x - 10 = x^2 - 3x + 11$

$2x^2 - x^2 + x + 3x - 10 - 11 = 0$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$\Rightarrow x = -7$ or $x = 3$

when $x = -7$

$y = (-7)^2 - 3(-7) + 11$

$y = 49 + 21 + 11 = 81$

when $x = 3$

$y = 3^2 - 3(3) + 11 = 11$

Points of intersection are

$(-7, 81)$ and $(3, 11)$

2)

i) $x^2 - 8x + 25$

$= (x-4)^2 + 25 - 16$

$= (x-4)^2 + 9$

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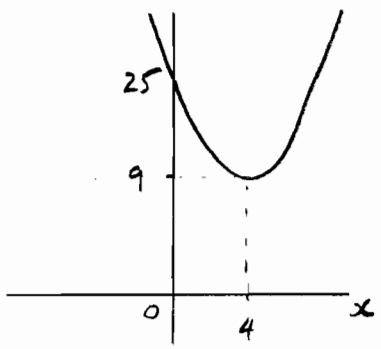
ii) Min point $(4, 9)$



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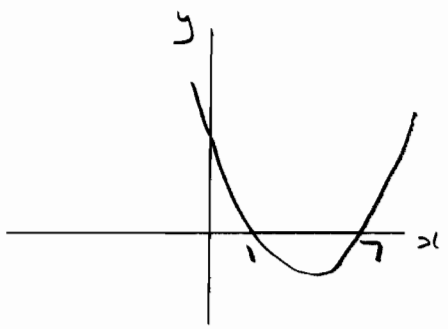
iii) $x^2 - 8x + 25 > 18$

$x^2 - 8x + 25 - 18 > 0$

$x^2 - 8x + 7 > 0$

$(x-7)(x-1) > 0$

Sketch $y = (x-7)(x-1)$



Solution

$x < 1$

or $x > 7$

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iv) $y = x^2 - 8x + 25$

Translate by $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$ gives

$y = x^2 - 8x + 25 - 20$

$y = x^2 - 8x + 5$

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3) i) $x^2 - 7x + 6$

$= \left(x - \frac{7}{2}\right)^2 + 6 - \frac{49}{4}$

$= \left(x - \frac{7}{2}\right)^2 - \frac{25}{4}$

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ii) Min point $\left(\frac{7}{2}, -\frac{25}{4}\right)$

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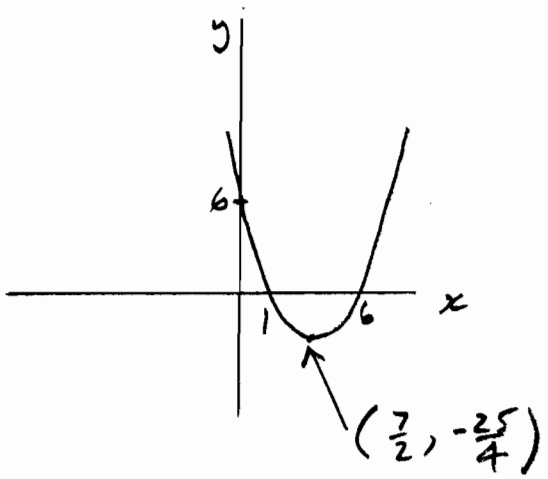
iii $y = x^2 - 7x + 6$

when $x = 0$, $y = 6$

when $y = 0$, $x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$x = 6$ or $x = 1$



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3 cont iv)

$y = x^2 - 7x + 6$ (1)

$y = x^2 - 3x + 4$ (2)

Sub for y in (1)

$x^2 - 3x + 4 = x^2 - 7x + 6$

$x^2 - 3x + 4 - x^2 + 7x - 6 = 0$

$4x - 2 = 0$

$4x = 2$

$x = \frac{1}{2}$

So only 1 solution which occurs at $x = \frac{1}{2}$

Not asked for y coord at point of intersection

$4x^2 - 6x - 18x + 27 = 0$

$2x(2x-3) - 9(2x-3) = 0$

$(2x-9)(2x-3) = 0$

$\Rightarrow 2x - 9 = 0$

$2x = 9$

$x = \frac{9}{2}$

or $2x - 3 = 0$

$2x = 3$

$x = \frac{3}{2}$

Solution $x = \frac{9}{2}$

$x = \frac{3}{2}$

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4) i) $4x^2 - 24x + 27$

$= 4\left(x^2 - 6x + \frac{27}{4}\right)$

$= 4\left((x-3)^2 + \frac{27}{4} - 9\right)$

$= 4(x-3)^2 + 27 - 36$

$= 4(x-3)^2 - 9$

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ii) Min point $(3, -9)$

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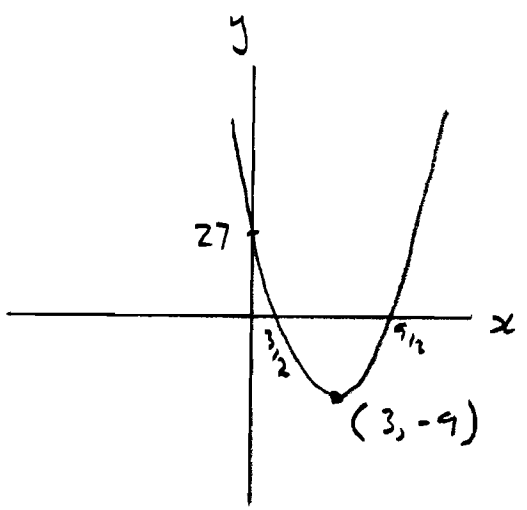
iii) $4x^2 - 24x + 27 = 0$

$4 \times 27 = 108$

Factors of 108 adding to -24 are -6 and -18

iv)



5) i) $x^2 - 5x + 8$

$= \left(x - \frac{5}{2}\right)^2 + 8 - \frac{25}{4}$

$= \left(x - \frac{5}{2}\right)^2 + \frac{7}{4}$

Minimum value $= \frac{7}{4}$

$\therefore > 0$ for all values of x

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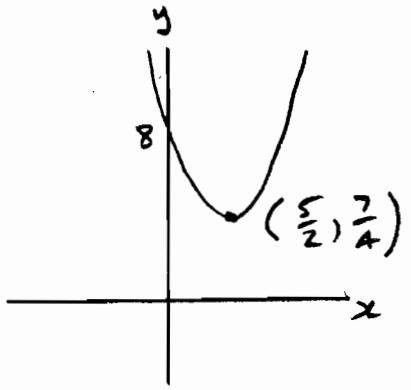
ii)



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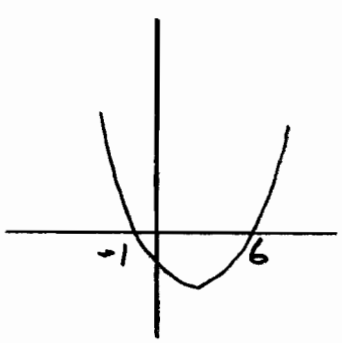
iii) $x^2 - 5x + 8 > 14$

$x^2 - 5x + 8 - 14 > 0$

$x^2 - 5x - 6 > 0$

$(x - 6)(x + 1) > 0$

Sketch $y = (x - 6)(x + 1)$



Solution $x > 6$

or $x < -1$

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iv) $f(x) = x^2 - 5x + 8$

$y = f(x) - 10$

is a translation of graph by $\begin{pmatrix} 0 \\ -10 \end{pmatrix}$

Min Point will be

$\left(\frac{5}{2}, \frac{7}{4} - 10\right)$

$= \left(\frac{5}{2}, -\frac{33}{4}\right)$

Which is below x-axis

So yes graph crosses the x-axis.