MEI CORE1 COORD GEOMETRY EXERCISE 2C

1) i) $x = 7$

ii) $y = 5$

iii) Passes through (0,0) and (2,4)

$\therefore$ gradient $= 2$

$y = 2x$

iv) Passes through (0,2) and (2,0)

$\therefore \quad x + y = 2$

$y = 2 - x$

v) Passes through (−4 −2) and (0,−3)

Gradient $\dfrac{-2 - -3}{-4 - 0} = \dfrac{+1}{-4}$

Gradient $= -\frac{1}{4}$

Using $y - y_1 = m(x - x_1)$

$y - -3 = -\frac{1}{4}(x - 0)$

$y + 3 = -\frac{1}{4}x$

$y = -\frac{1}{4}x - 3$

vi) Passes through (0,0) and (4,4)

$y = x$

vii) $x = -4$

viii) $y = -4$

ix) Passes through (−4,2) and (0,0)

Gradient $= \dfrac{2 - 0}{-4 - 0} = \dfrac{2}{-4} = -\dfrac{1}{2}$

$y = -\frac{1}{2}x$

x) Passes through (0,4) and (6,2)

Gradient $= \dfrac{4 - 2}{0 - 6} = \dfrac{2}{-6} = -\dfrac{1}{3}$

Using $y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{3}(x - 0)$

$y - 4 = -\frac{1}{3}x$

$y = -\frac{1}{3}x + 4$

2) i) Parallel to $y = 2x$ thro (1,5)

$\Rightarrow$ gradient $= 2$

Using $y - y_1 = m(x - x_1)$

$y - 5 = 2(x - 1)$

$y - 5 = 2x - 2$

$y = 2x - 2 + 5$

$y = 2x + 3$

ii) Parallel to $y = 3x - 1$ thro (0,0)

$\Rightarrow$ gradient $= 3$

Using $y - y_1 = m(x - x_1)$

$y - 0 = 3(x - 0) \Rightarrow y = 3x$

2 iii) Parallel to $2x+y-3=0$ thro $(-4,5)$

$\Rightarrow y = -2x+3$

$\Rightarrow$ gradient $= -2$

Using $y - y_1 = m(x - x_1)$

$y - 5 = -2(x - 4)$

$y - 5 = -2x + 8$

$y = -2x + 13$

2 iv) Parallel to $3x - y - 1 = 0$ through $(4,-2)$

$3x - 1 = y$

$\Rightarrow$ Gradient $= 3$

Using $y - y_1 = m(x - x_1)$

$y - -2 = 3(x - 4)$

$y + 2 = 3x - 12$

$y = 3x - 12 - 2$

$y = 3x - 14$

2 v) Parallel to $2x + 3y = 4$ thro $(2,2)$

$2x + 3y = 4$

$3y = -2x + 4$

$y = -\frac{2}{3}x + \frac{4}{3}$

Gradient $= -\frac{2}{3}$

Using $y - y_1 = m(x - x_1)$

$y - 2 = -\frac{2}{3}(x - 2)$

$y - 2 = -\frac{2}{3}x + \frac{4}{3}$

$y = -\frac{2}{3}x + \frac{4}{3} + 2$

$y = -\frac{2}{3}x + \frac{10}{3}$

2 vi) Parallel to $2x - y - 8 = 0$ thro $(-1,-5)$

$2x - 8 = y$

Gradient $= 2$

Using $y - y_1 = m(x - x_1)$

$y - -5 = 2(x - -1)$

$y + 5 = 2(x + 1)$

$y + 5 = 2x + 2$

$y = 2x + 2 - 5$

$y = 2x - 3$

3) i) $\perp$ to $y = 3x$ thro $(0,0)$

Require gradient $= -\frac{1}{3}$

Using $y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{3}(x - 0)$

$y = -\frac{1}{3}x$

3 ii) $\perp$ to $y = 2x + 3$ thro $(2,-1)$

Require gradient $= -\frac{1}{2}$

Using $y - y_1 = m(x - x_1)$

3ii) cont

$y - -1 = -\frac{1}{2}(x-2)$

$y+1 = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x + 1 - 1$

$y = -\frac{1}{2}x$

3iii) $\perp$ to $2x+y=4$ thro $(3,1)$

$y = -2x+4$

Require gradient $= +\frac{1}{2}$

Using $y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2}(x-3)$

$y - 1 = \frac{1}{2}x - \frac{3}{2}$

$y = \frac{1}{2}x - \frac{3}{2} + 1$

$y = \frac{1}{2}x - \frac{1}{2}$

3iv) $\perp$ to $2y = x+5$ thro $(-1,4)$

$y = \frac{1}{2}x + \frac{5}{2}$

Require gradient $= -2$

Using $y - y_1 = m(x - x_1)$

$y - 4 = -2(x - -1)$

$y - 4 = -2(x+1)$

$y - 4 = -2x - 2$

$y = -2x - 2 + 4$

$y = -2x + 2$

3v) $\perp$ to $2x+3y=4$ thro $(5,-1)$

$3y = -2x + 4$

$y = -\frac{2}{3}x + \frac{4}{3}$

Require gradient $= +\frac{3}{2}$

Using $y - y_1 = m(x - x_1)$

$y - -1 = \frac{3}{2}(x-5)$

$y + 1 = \frac{3}{2}x - \frac{15}{2}$

$y = \frac{3}{2}x - \frac{15}{2} - 1$

$y = \frac{3}{2}x - \frac{17}{2}$

3vi) $\perp$ to $4x - y + 1 = 0$ thro $(0,6)$

$4x + 1 = y$

Require gradient $= -\frac{1}{4}$

Using $y - y_1 = m(x - x_1)$

$y - 6 = -\frac{1}{4}(x - 0)$

$y - 6 = -\frac{1}{4}x$

$y = -\frac{1}{4}x + 6$

4i) $A(0,0)$ $B(4,3)$

Using $\dfrac{y - y_1}{y_1 - y_2} = \dfrac{x - x_1}{x_1 - x_2}$

$\dfrac{y - 0}{0 - 3} = \dfrac{x - 0}{0 - 4}$

4 i) cont

$$\frac{y}{-3} = \frac{x}{-4}$$

$$-4y = -3x$$

$$y = \frac{-3x}{-4}$$

$$y = \frac{3x}{4}$$

4 ii) A(2, −1) B(3, 0)

Using $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

$$\frac{y--1}{-1-0} = \frac{x-2}{2-3}$$

$$\frac{y+1}{-1} = \frac{x-2}{-1}$$

$$y+1 = x-2$$

$$y = x-2-1$$

$$y = x-3$$

4 iii) A(2, 7) B(2, −3)

Using $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

$$\frac{y-7}{7--3} = \frac{x-2}{2-2}$$

$$\frac{y-7}{10} = \frac{x-2}{0}$$

Cannot $\div 0$ Notice $x = 2$ for both A and B

Line is $x = 2$

4 iv) A(3, 5) B(5, −1)

Using $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

$$\frac{y-5}{5--1} = \frac{x-3}{3-5}$$

$$\frac{y-5}{6} = \frac{x-3}{-2}$$

$$-2(y-5) = 6(x-3)$$

$$-2y+10 = 6x-18$$

$$-2y = 6x-18-10$$

$$-2y = 6x-28$$

$$y = \frac{6x}{-2} - \frac{28}{-2}$$

$$y = -3x+14$$

4 v) A(−2, 4) B(5, 3)

Using $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

$$\frac{y-4}{4-3} = \frac{x--2}{-2-5}$$

$$\frac{y-4}{1} = \frac{x+2}{-7}$$

$$-7(y-4) = x+2$$

$$-7y+28 = x+2$$

$$-7y = x+2-28$$

4v) Cont

$-7y = x - 26$

$y = -\frac{1}{7}x + \frac{26}{7}$

4vi) $A(-4,-2)$ $B(3,-2)$

Using $\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

$\frac{y--2}{-2--2} = \frac{x--4}{-4-3}$

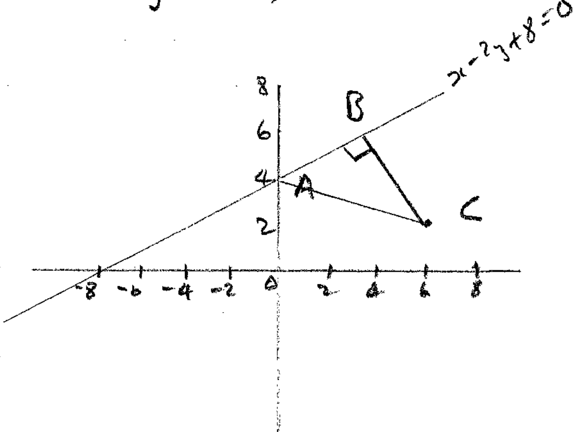
$\frac{y+2}{0} = \frac{x+4}{-7}$

Cannot divide by 0
$y = -2$ for both A and B

Line is $y = -2$

5) i) AB part of $x - 2y + 8 = 0$

When $x = 0$, $y = +4$
when $y = 0$, $x = -8$



A on y axis $\therefore$ $A(0,4)$

ii) $A(0,4)$ $C(6,2)$

To find AC use

$\frac{y-y_1}{y_1-y_2} = \frac{x-x_1}{x_1-x_2}$

$\frac{y-4}{4-2} = \frac{x-0}{0-6}$

$\frac{y-4}{2} = \frac{x}{-6}$

$-6(y-4) = 2x$

$-6y + 24 = 2x$

$-6y = 2x - 24$

AC $y = -\frac{1}{3}x + 4$

BC is $\perp$ to $x - 2y + 8 = 0$
$x + 8 = 2y$
$\frac{1}{2}x + 4 = y$

$\therefore$ BC has gradient -2 and passes through $C(6,2)$

Using $y - y_1 = m(x - x_1)$

$y - 2 = -2(x - 6)$

$y - 2 = -2x + 12$

$y = -2x + 14$

iii) To find B solve

$\left.\begin{aligned} x - 2y + 8 &= 0 \\ y &= -2x + 14 \end{aligned}\right\}$ ① ②

Subst for y in ①

$x - 2(-2x + 14) + 8 = 0$

$x + 4x - 28 + 8 = 0$

$5x - 20 = 0$

5 iii) cont.

$5x = 20$

$x = 4$

Subst for x in (2)

$y = -2\times4 + 14$

$y = -8 + 14$

$y = 6$

$\therefore$ B is point $(4, 6)$

We now have

A $(0, 4)$
B $(4, 6)$
C $(6, 2)$

Length AB $= \sqrt{(4-0)^2 + (6-4)^2}$

$= \sqrt{4^2 + 2^2}$

$= \sqrt{20}$ or $2\sqrt{5}$

Length BC $= \sqrt{(6-4)^2 + (2-6)^2}$

$= \sqrt{2^2 + (-4)^2}$

$= \sqrt{20}$ or $2\sqrt{5}$

Area of $\triangle = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times \sqrt{20} \times \sqrt{20}$

$= \frac{1}{2} \times 20 = 10$ units^2

iv) Could calculate area of $\triangle ABC$ by $\frac{1}{2} \times$ base AC $\times \perp$ to B

Length AC $= \sqrt{(6-0)^2 + (2-4)^2}$

$= \sqrt{6^2 + (-2)^2}$

$= \sqrt{40}$

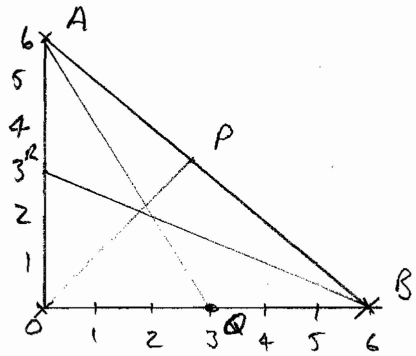
$\therefore \frac{1}{2} \times \sqrt{40} \times$ length of $\perp$ to B $= 10$

length of $\perp$ to B $= \dfrac{20}{\sqrt{40}}$

$= \dfrac{2\times10}{2\sqrt{10}}$

$= \sqrt{10}$ units

6) i)



ii) Midpoint P of AB $= (3, 3)$

Eqn of OP is $y = x$

Midpoint Q of OB is $(3, 0)$

Find eqn of AQ A$(0,6)$ Q$(3,0)$

Using $\dfrac{y-y_1}{y_1-y_2} = \dfrac{x-x_1}{x_1-x_2}$

$\dfrac{y-6}{6-0} = \dfrac{x-0}{0-3}$

$\dfrac{y-6}{6} = \dfrac{x}{-3}$

6 ii) cont

$-3(y-6) = 6x$

$-3y + 18 = 6x$

$-3y = 6x - 18$

$y = -2x + 6$

Midpoint R of OA $= (0,3)$

R$(0,3)$ and B$(6,0)$

Eqn of RB

using $\dfrac{y-y_1}{y_1-y_2} = \dfrac{x-x_1}{x_1-x_2}$

$\dfrac{y-3}{3-0} = \dfrac{x-0}{0-6}$

$\dfrac{y-3}{3} = \dfrac{x}{-6}$

$-6(y-3) = 3x$

$-6y + 18 = 3x$

$-6y = 3x - 18$

$y = -\frac{1}{2}x + 3$

6 iii)

OP $y = x$

when $x = 2$, $y = 2$ $\therefore (2,2)$ on line

AQ $y = -2x + 6$

when $x = 2$, $y = -2\times 2 + 6$
$= -4 + 6 = 2$

$\therefore (2,2)$ on line

RB $y = -\frac{1}{2}x + 3$

when $x = 2$

$y = -\frac{1}{2}\times 2 + 3$

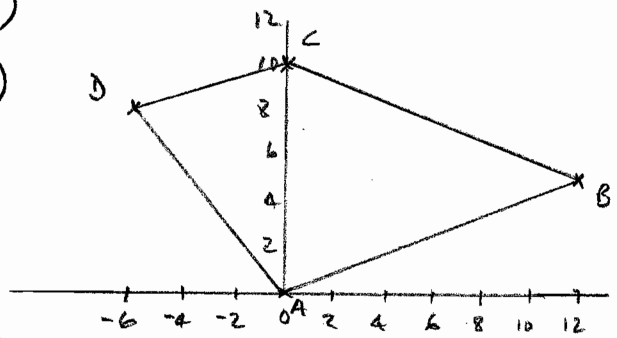
$= -1 + 3 = 2$

$\therefore (2,2)$ on line

And all 3 medians are concurrent

7)

i)



ii)

A$(0,0)$
B$(12,5)$
C$(0,10)$
D$(-6,8)$

Gradient of AB $= \dfrac{5-0}{12-0} = \dfrac{5}{12}$

Gradient of BC $= \dfrac{10-5}{0-12} = \dfrac{5}{-12} = -\dfrac{5}{12}$

Gradient of CD $= \dfrac{8-10}{-6-0} = \dfrac{-2}{-6} = +\dfrac{1}{3}$

Gradient of DA $= \dfrac{8-0}{-6-0} = \dfrac{8}{-6} = -\dfrac{4}{3}$

iii)

Length AB $= \sqrt{(12-0)^2 + (5-0)^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169} = 13$ units

7iii) cont

Length BC $= \sqrt{(12-0)^2 + (5-0)^2}$

$= \sqrt{144 + 25}$

$= 13$ units

Length CD $= \sqrt{(0--6)^2 + (10-8)^2}$

$= \sqrt{36 + 4}$

$= \sqrt{40}$ units

Length DA $= \sqrt{(-6-0)^2 + (8-0)^2}$

$= \sqrt{36 + 64}$

$= 10$ units

7iv)

Eqn of AB
Passes through $(0,0)$ with grad $\frac{5}{12}$

Using $y - y_1 = m(x - x_1)$

$y - 0 = \frac{5}{12}(x - 0)$

$y = \frac{5}{12}x$

Eqn of BC
Passes through $(0,10)$ gradient $-\frac{5}{12}$

Using $y - y_1 = m(x - x_1)$

$y - 10 = -\frac{5}{12}(x - 0)$

$y - 10 = -\frac{5}{12}x$

$y = -\frac{5}{12}x + 10$

Eqn of CD
Passes through $(0,10)$ gradient $\frac{1}{3}$

Using $y - y_1 = m(x - x_1)$

$y - 10 = \frac{1}{3}(x - 0)$

$y - 10 = \frac{1}{3}x$

$y = \frac{1}{3}x + 10$

Eqn of DA
Passes through $(0,0)$ gradient $-\frac{4}{3}$

Using $y - y_1 = m(x - x_1)$

$y - 0 = -\frac{4}{3}(x - 0)$

$y = -\frac{4}{3}x$

7v) Area of quadrilateral

$=$ Area of $\triangle ABC$ + Area of $\triangle ADC$

Taking AC to be base of both $\triangle$s

Area $= \frac{1}{2} \times 10 \times 12 + \frac{1}{2} \times 10 \times 6$

$= 60 + 30$

$= 90$ units^2