1) Given $2^x = 32 \iff x = \log_2 32$

i) $3^x = 9 \iff x = \log_3 9$

$\qquad x = 2$

ii) $4^x = 64 \iff x = \log_4 64$

$\qquad x = 3$

iii) $2^x = \frac{1}{4} \iff x = \log_2 \frac{1}{4}$

$\qquad x = -2$

iv) $5^x = \frac{1}{5} \iff x = \log_5 \frac{1}{5}$

$\qquad x = -1$

v) $7^x = 1 \iff x = \log_7 1$

$\qquad x = 0$

vi) $16^x = 2 \iff x = \log_{16} 2$

$\qquad x = \frac{1}{4}$

2)
i) $y = \log_3 9 \iff 3^y = 9$

$\qquad y = 2$

ii) $y = \log_5 125 \iff 5^y = 125$

$\qquad y = 3$

iii) $y = \log_2 16 \iff 2^y = 16$

$\qquad y = 4$

iv) $y = \log_6 1 \iff 6^y = 1$

$\qquad y = 0$

2 v) $y = \log_{64} 8 \iff 64^y = 8$

$\qquad y = \frac{1}{2}$

vi) $y = \log_5 \frac{1}{25} \iff 5^y = \frac{1}{25}$

$\qquad y = -2$

3)
i) $\log_{10} 10000 = 4$

ii) $\log_{10} \left(\frac{1}{10000}\right) = -4$

iii) $\log_{10} \sqrt{10} = \frac{1}{2}$

iv) $\log_{10} 1 = 0$

v) $\log_3 81 = 4$

vi) $\log_3 \left(\frac{1}{81}\right) = -4$

vii) $\log_3 \sqrt{27} = \frac{3}{2}$

viii) $\log_3 \sqrt[4]{3} = \frac{1}{4}$

ix) $\log_4 2 = \frac{1}{2}$

x) $\log_5 \left(\frac{1}{125}\right) = -3$

4)
i) $\log 5 + \log 2 = \log 10$

ii) $\log 6 - \log 3 = \log \left(\frac{6}{3}\right)$

$\qquad\qquad\qquad = \log 2$

iii) $2 \log 6 = \log 6^2 = \log 36$

iv) $-\log 7 = \log 7^{-1}$

$\qquad\qquad = \log \left(\frac{1}{7}\right)$

4 v) $\frac{1}{2} \log 9 = \log 9^{\frac{1}{2}}$

$= \log 3$

vi) $\frac{1}{4} \log 16 + \log 2$

$= \log 16^{\frac{1}{4}} + \log 2$

$= \log 2 + \log 2 = \log 4$

vii) $\log 5 + 3 \log 2 - \log 10$

$= \log 5 + \log 2^3 - \log 10$

$= \log 5 + \log 8 - \log 10$

$= \log \left(\frac{5 \times 8}{10} \right) = \log 4$

viii) $\log 12 - 2 \log 2 - \log 9$

$= \log 12 - \log 4 - \log 9$

$= \log \left(\frac{12}{4 \times 9} \right) = \log \left(\frac{1}{3} \right)$

ix) $\frac{1}{2} \log 16 + 2 \log \left(\frac{1}{2} \right)$

$= \frac{1}{2} \log 4 + \log \left(\frac{1}{2} \right)^2$

$= \log 2 + \log \left(\frac{1}{4} \right)$

$= \log \left(2 \times \frac{1}{4} \right) = \log \left(\frac{1}{2} \right)$

x) $2 \log 4 + \log 9 - \frac{1}{2} \log 144$

$= \log 16 + \log 9 - \log 12$

$= \log \left(\frac{16 \times 9}{12} \right) = \log 12$

5)

i) $\log x^2 = 2 \log x$

ii) $\log x^5 - 2 \log x$

$= 5 \log x - 2 \log x$

$= 3 \log x$

iii) $\log \sqrt{x} = \frac{1}{2} \log x$

iv) $\log x^{3/2} + \log \sqrt[3]{x}$

$= \frac{3}{2} \log x + \frac{1}{3} \log x$

$= \frac{11}{6} \log x$

v) $3 \log x + \log x^3$

$= 3 \log x + 3 \log x = 6 \log x$

vi) $\log \left(\sqrt{x} \right)^5 = \frac{5}{2} \log x$

6)

$2 \log_{10} x - \log_{10} 7$

$= \log_{10} x^2 - \log_{10} 7$

$= \log \left(\frac{x^2}{7} \right)$

Solve

$2 \log_{10} x - \log_{10} 7 = \log_{10} 63$

$\Rightarrow \log_{10} \left(\frac{x^2}{7} \right) = \log_{10} 63$

$\Rightarrow \frac{x^2}{7} = 63$

6cont) $x^2 = 441$

$\Rightarrow x = 21$

(Note $x \neq -21$ since $\log_{10} x$ not defined for $x \leq 0$)

7)

i) $2^x = 1,000,000$

$x \log_{10} 2 = \log_{10} 1,000,000$

$x = \dfrac{6}{\log_{10} 2}$

$x = 19.93$ to 2 dp

ii)

$2^x = 0.001$

$x \log_{10} 2 = \log_{10} 0.001$

$x = \dfrac{-3}{\log_{10} 2}$

$x = -9.97$ to 2 dp

iii)

$1.08^x = 2$

$x \log_{10} 1.08 = \log_{10} 2$

$x = \dfrac{\log_{10} 2}{\log_{10} 1.08}$

$x = 9.01$ to 2 dp.

iv)

$1.1^x = 100$

$x \log_{10} 1.1 = \log_{10} 100$

$x = \dfrac{2}{\log_{10} 1.1}$

$x = 48.32$ to 2 dp

7v)

$0.99^x = 0.000001$

$x \log_{10} 0.99 = \log_{10} 0.000001$

$x = \dfrac{\log_{10} 0.000001}{\log_{10} 0.99}$

$x = 1374.63$ to 2 dp

8)

GP $a = 5$, $r = 7$

$ar^{m-1} = 28,824,005$

$5 \times 7^{m-1} = 28,824,005$

$7^{m-1} = \dfrac{28,824,005}{5}$

$7^{m-1} = 5764801$

$(m-1) \log_{10} 7 = \log_{10} 5764801$

$m - 1 = \dfrac{\log_{10} 5764801}{\log_{10} 7}$

$m - 1 = 8$

$m = 9$

9)

i) $-1, 2, -4, 8, \ldots \ldots -16777216$

$a = -1, \quad r = -2$

n^{th} term $= -16777216$

$ar^{n-1} = -16777216$

$-1(-2)^{n-1} = -16777216$

$(-2)^{n-1} = 16777216$

$\Rightarrow 2^{n-1} = 16777216$

$\Rightarrow (n-1)\log 2 = \log 16777216$

$(n-1) = \dfrac{\log 16777216}{\log 2}$

$n - 1 = 24$

$n = 25$

9ii)

$0.1, 0.3, 0.9, \ldots 4304672.1$

$a = 0.1, \quad r = 3$

n^{th} term
$\quad ar^{n-1} = 4304672.1$

$0.1 \times 3^{n-1} = 4304672.1$

$3^{n-1} = 43046721$

$(n-1)\log_{10} 3 = \log_{10} 43046721$

$n - 1 = \dfrac{\log_{10} 43046721}{\log_{10} 3}$

$n - 1 = 16 \quad \Rightarrow n = 17$

10)
$1.07^{n} = 2$

$n \log_{10} 1.07 = \log_{10} 2$

$n = \dfrac{\log_{10} 2}{\log_{10} 1.07}$

$n = 10.24$

Would double in 11th year