MEI FP1 MATRICES EXERCISE 1G

1) i) $\begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix} = 2 - -3 = 5$

$$\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}^{-1} = \frac{1}{5}\begin{pmatrix} 1 & -3 \\ 1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}$$

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ii) $$\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{5} + \frac{9}{5} \\ \frac{1}{5} - \frac{6}{5} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

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2) i) $$\begin{aligned} 3x - y &= 2 \\ 2x + 3y &= 5 \end{aligned}$$

$$\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix} = 9 + 2 = 11$$

$$\begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}^{-1} = \frac{1}{11}\begin{pmatrix} 3 & 1 \\ -2 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{11} & \frac{1}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{3}{11} & \frac{1}{11} \\ -\frac{2}{11} & \frac{3}{11} \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{6}{11} + \frac{5}{11} \\ -\frac{4}{11} + \frac{15}{11} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

2ii) $$\begin{aligned} 3x + 2y &= 4 \\ x - 2y &= 4 \end{aligned}$$

$$\begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

$$\begin{vmatrix} 3 & 2 \\ 1 & -2 \end{vmatrix} = -6 - 2 = -8$$

$$\begin{pmatrix} 3 & 2 \\ 1 & -2 \end{pmatrix}^{-1} = -\frac{1}{8}\begin{pmatrix} -2 & -2 \\ -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{8} & -\frac{3}{8} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{8} & -\frac{3}{8} \end{pmatrix}\begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

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2ii cont) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{4}{8} & -\frac{12}{8} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

2iii)

$x + 3y = 11$
$2x - y = 1$

$\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 1 \end{pmatrix}$

$\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -1 - 6 = -7$

$\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}^{-1} = -\frac{1}{7}\begin{pmatrix} -1 & -3 \\ -2 & 1 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{7} & \frac{3}{7} \\ \frac{2}{7} & -\frac{1}{7} \end{pmatrix}\begin{pmatrix} 11 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{11}{7} + \frac{3}{7} \\ \frac{22}{7} - \frac{1}{7} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

2iv)

$3x - 2y = 9$
$x - 4y = -2$

$\begin{pmatrix} 3 & -2 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -2 \end{pmatrix}$

$\begin{vmatrix} 3 & -2 \\ 1 & -4 \end{vmatrix} = -12 + 2 = -10$

$\begin{pmatrix} 3 & -2 \\ 1 & -4 \end{pmatrix}^{-1} = -\frac{1}{10}\begin{pmatrix} -4 & 2 \\ -1 & 3 \end{pmatrix}$

$= \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{1}{10} & -\frac{3}{10} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{1}{10} & -\frac{3}{10} \end{pmatrix}\begin{pmatrix} 9 \\ -2 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{18}{5} + \frac{2}{5} \\ \frac{9}{10} + \frac{6}{10} \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1.5 \end{pmatrix}$

3) i)

$3x + 5y = 17$
$2x + 4y = 11$

$\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 17 \\ 11 \end{pmatrix}$

$\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} = 12 - 10 = 2$

$\begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -5 \\ -2 & 3 \end{pmatrix}$

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3i) cont

$$= \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & 3/2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -\frac{5}{2} \\ -1 & 3/2 \end{pmatrix} \begin{pmatrix} 17 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 34 - \frac{55}{2} \\ -17 + \frac{33}{2} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$$

Non-parallel lines unique solution

3ii)

$$3x + 6y = 12$$
$$2x + 4y = 15$$

$$\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 15 \end{pmatrix}$$

$$\begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} = 12 - 12 = 0$$

Reduce to $x + 2y = 4$
$x + 2y = 7.5$

Inconsistent
Parallel lines no solutions.

3iii)

$$6x - 3y = 12$$
$$2x - y = 4$$

$$\begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$$

$$\begin{vmatrix} 6 & -3 \\ 2 & -1 \end{vmatrix} = -6 + 6 = 0$$

Reduce to $2x - y = 4$
$2x - y = 4$

Same line infinitely many solutions

$x = \lambda$, $y = 2\lambda - 4$

3iv)

$$8x - 4y = 11$$
$$y = 2x - 4$$

$$8x - 4y = 11$$
$$2x - y = 4$$

$$\begin{pmatrix} 8 & -4 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 4 \end{pmatrix}$$

$$\begin{vmatrix} 8 & -4 \\ 2 & -1 \end{vmatrix} = -8 + 8 = 0$$

Reduce to $2x - y = \frac{11}{4}$
$2x - y = 4$

Inconsistent
Parallel lines no solution.

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4)

$$2x + ky = 3$$
$$kx + 8y = 6$$

$$\begin{pmatrix} 2 & k \\ k & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$$\begin{vmatrix} 2 & k \\ k & 8 \end{vmatrix} = 16 - k^2$$

$= 0$ when $k = \pm 4$

When $k = 4$

$$2x + 4y = 3$$
$$4x + 8y = 6$$

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4 cont) Coincident
Both lines reduce to $x + 2y = 3/2$
Infinitely many solutions

$x = \lambda$, $y = -\frac{\lambda}{2} + \frac{3}{4}$

When $k = -4$

$$2x - 4y = 3$$
$$-4x + 8y = 6$$

Reduce to $x - 2y = 3/2$
$x - 2y = -3/2$

Inconsistent
Parallel lines no solution.

5)

$$\underline{AB} = \begin{pmatrix} 5 & -2 & k \\ 3 & -4 & -5 \\ -2 & 3 & 4 \end{pmatrix}\begin{pmatrix} -1 & 3k+8 & 4k+10 \\ -2 & 2k+20 & 3k+25 \\ 1 & -11 & -14 \end{pmatrix}$$

$$= \begin{pmatrix} -5+4+k & 15k+40 \\ & -4k-40 \\ & -11k \\ -3+8-5 & 9k+24 \\ & -8k-80 \\ & +55 \\ 2-6+4 & -6k-16 \\ & +6k+60 \\ & -44 \end{pmatrix} \quad \begin{matrix} 20k+50 \\ -6k-50 \\ -14k \\ 12k+30 \\ -12k-100 \\ +70 \\ -8k-20 \\ +9k+75 \\ -56 \end{matrix}$$

$$= \begin{pmatrix} k-1 & 0 & 0 \\ 0 & k-1 & 0 \\ 0 & 0 & k-1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{k-1}\begin{pmatrix} -1 & 3k+8 & 4k+10 \\ -2 & 2k+20 & 3k+25 \\ 1 & -11 & -14 \end{pmatrix}$$

$k \neq 1$

5ii) a)
$$\begin{pmatrix} 5 & -2 & 8 \\ 3 & -4 & -5 \\ -2 & 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 28 \\ 0 \\ m \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{pmatrix} -1 & 32 & 42 \\ -2 & 36 & 49 \\ 1 & -11 & -14 \end{pmatrix}\begin{pmatrix} 28 \\ 0 \\ m \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{7}\begin{pmatrix} -28 + 42m \\ -56 + 49m \\ 28 - 14m \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6m - 4 \\ 7m - 8 \\ -2m + 4 \end{pmatrix}$$

5ii) b)
$$\begin{pmatrix} 5 & -2 & 1 \\ 3 & -4 & -5 \\ -2 & 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 28 \\ 0 \\ 4 \end{pmatrix}$$

$$5x - 2y + z = 28 \quad (1)$$
$$3x - 4y - 5z = 0 \quad (2)$$
$$-2x + 3y + 4z = 4 \quad (3)$$

$5\times(1)+(2)$ $\quad 28x - 14y = 140 \quad (4)$
$4\times(1)-(3)$ $\quad 22x - 11y = 108 \quad (5)$

Reduce to $2x - y = 10$
$2x - y = 108/11$

No solution.

5ii) c)
$$\begin{pmatrix} 5 & -2 & 1 \\ 3 & -4 & -5 \\ -2 & 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 28 \\ 0 \\ 2 \end{pmatrix}$$

$$5x - 2y + z = 28 \quad (1)$$
$$3x - 4y - 5z = 0 \quad (2)$$
$$-2x + 3y + 4z = 2 \quad (3)$$

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Scan. $5\times①+②$ $28x-14y=140$ ④
$4\times①-③$ $22x-11y=110$ ⑤

Both reduce to $2x-y=10$
$2x-y=10$

Infinitely many solutions.

$x=\lambda$, $y=2\lambda-10$

Since $3x-4y-5z=0$

$3\lambda-8\lambda+40-5z=0$
$-5\lambda+40=5z$

$z=8-\lambda$

Solution $x=\lambda$
$y=2\lambda-10$
$z=8-\lambda$

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6)

$\underline{A}\,\underline{B}=\begin{pmatrix}1&-2&0\\0&2&-2\\a&0&3\end{pmatrix}\begin{pmatrix}6&6&4\\k&3&2\\k&k&2\end{pmatrix}$

i)

$=\begin{pmatrix}6-2k&0&0\\0&6-2k&0\\6a+3k&6a+3k&4a+6\end{pmatrix}$

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ii)

Need $4a+6=6-2k$
$4a+2k=0$

Also need $6a+3k=0$

Both satisfied if
$2a+k=0$

In which case matrix is

$\begin{pmatrix}6-2k&0&0\\0&6-2k&0\\0&0&6-2k\end{pmatrix}$

$=(6-2k)\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

6iii)

$\underline{A}^{-1}=\frac{1}{6-2k}\begin{pmatrix}6&6&4\\k&3&2\\k&k&2\end{pmatrix}$

But $k=-2a$.

$\therefore\ \underline{A}^{-1}=\frac{1}{6+4a}\begin{pmatrix}6&6&4\\-2a&3&2\\-2a&-2a&2\end{pmatrix}$

$\underline{B}^{-1}=\frac{1}{6-2k}\begin{pmatrix}1&-2&0\\0&2&-2\\a&0&3\end{pmatrix}$

$=\frac{1}{6-2k}\begin{pmatrix}1&-2&0\\0&2&-2\\-\frac{k}{2}&0&3\end{pmatrix}$

6iv)

$\begin{pmatrix}1&-2&0\\0&2&-2\\a&0&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2\\-2\\3\end{pmatrix}$

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6 cont

iv) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{6+4a}\begin{pmatrix} 6 & 6 & 4 \\ -2a & 3 & 2 \\ -2a & -2a & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{(6+4a)}\begin{pmatrix} 12 \\ -4a \\ 6 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{12}{6+4a} \\ \frac{-4a}{6+4a} \\ \frac{6}{6+4a} \end{pmatrix}$

6 v) $(AB)^{-1} = B^{-1}A^{-1}$

$= \frac{1}{(6-2k)}\cdot\frac{1}{(6+4a)}\begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & -2 \\ -\frac{k}{2} & 0 & 3 \end{pmatrix}\begin{pmatrix} 6 & 6 & 4 \\ -2a & 3 & 2 \\ -2a & -2a & 2 \end{pmatrix}$

$= \frac{1}{(6-2k)(6+4a)}\begin{pmatrix} 6+4a & 0 & 0 \\ 0 & 6+4a & 0 \\ -3k-6a & -3k-6a & -2k+6 \end{pmatrix}$

7) i) $PQ = \begin{pmatrix} -2 & 26 & -16 \\ 1 & -11 & 7 \\ -1 & 21 & -13 \end{pmatrix}\begin{pmatrix} -2 & 1 & k \\ 3 & 5 & -1 \\ 5 & 8 & -2 \end{pmatrix}$

$= \begin{pmatrix} 2 & 0 & -2k+6 \\ 0 & 2 & k-3 \\ 0 & 0 & -k+5 \end{pmatrix}$

when $k = 3$

$PQ = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$\therefore Q^{-1} = \begin{pmatrix} -1 & 13 & -8 \\ \frac{1}{2} & -\frac{11}{2} & \frac{7}{2} \\ -\frac{1}{2} & \frac{21}{2} & -\frac{13}{2} \end{pmatrix}$

$Q\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 19 \\ 4 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & 13 & -8 \\ \frac{1}{2} & -\frac{11}{2} & \frac{7}{2} \\ -\frac{1}{2} & \frac{21}{2} & -\frac{13}{2} \end{pmatrix}\begin{pmatrix} 19 \\ 4 \\ 5 \end{pmatrix}$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -7 \\ 5 \\ 0 \end{pmatrix}$

iii) $\begin{pmatrix} -2 & 1 & 5 \\ 3 & 5 & -1 \\ 5 & 8 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 19 \\ 4 \\ 5 \end{pmatrix}$

$-2x + y + 5z = 19$ (1)
$3x + 5y - z = 4$ (2)
$5x + 8y - 2z = 5$ (3)

(1) + 5(2) $13x + 26y = 39$
$x + 2y = 3$ (4)

(2)×2 − (3) $x + 2y = 3$ (5)

Same eqn infinitely many solutions
$x = \lambda$, $y = \frac{3-\lambda}{2}$

Substitute (3) $5\lambda + 12\lambda - 2z = 5$
$z = \frac{17\lambda - 5}{2}$