1) $\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)^{15}$

i) $= \cos\frac{15\pi}{4} + j\sin\frac{15\pi}{4}$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j$

ii) $\left(\cos\frac{\pi}{3} + j\sin\frac{\pi}{3}\right)^{-8}$

$= \cos\left(-\frac{8\pi}{3}\right) + j\sin\left(-\frac{8\pi}{3}\right)$

$= \cos\left(-\frac{2\pi}{3}\right) + j\sin\left(-\frac{2\pi}{3}\right)$

$= -\frac{1}{2} - \frac{\sqrt{3}}{2}j$

iii) $\left(\cos\left(\frac{-\pi}{12}\right) + j\sin\left(\frac{-\pi}{12}\right)\right)^{10}$

$= \cos\left(-\frac{5\pi}{6}\right) + \sin\left(-\frac{5\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2} - \frac{1}{2}j$

iv) $\left(\cos\frac{7\pi}{8} - j\sin\frac{7\pi}{8}\right)^{6}$

$= \left(\cos\left(-\frac{7\pi}{8}\right) + j\sin\left(-\frac{7\pi}{8}\right)\right)^{6}$

$= \cos\left(-\frac{21\pi}{4}\right) + j\sin\left(-\frac{21\pi}{4}\right)$

$= \cos\left(-\frac{5\pi}{4}\right) + j\sin\left(-\frac{5\pi}{4}\right)$

$= \cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)$

$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j$

2) $(1 - \sqrt{3}j)^4$

i) $= \left(2\left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)\right)^4$

$= 16\left(\cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)\right)^4$

$= 16\left(\cos\left(\frac{-4\pi}{3}\right) + j\sin\left(\frac{-4\pi}{3}\right)\right)$

$= 16\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$

$= -8 + 8\sqrt{3}j$

ii) $(-2 + 2j)^7$

$= \left(\sqrt{8}\left(\frac{-2}{\sqrt{8}} + \frac{2}{\sqrt{8}}j\right)\right)^7$

$= \left(2\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j\right)\right)^7$

$1024\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right)^7$

$= 1024\sqrt{2}\left(\cos\left(\frac{21\pi}{4}\right) + j\sin\left(\frac{21\pi}{4}\right)\right)$

$= 1024\sqrt{2}\left(\cos\left(\frac{5\pi}{4}\right) + j\sin\left(\frac{5\pi}{4}\right)\right)$

$= 1024\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)$

$= -1024 - 1024j$

iii) $(0.6 + 0.8j)^{-5}$

$0.6^2 + 0.8^2 = 1$

$\cos^{-1}0.6 = 0.927295218$ rads say α

$= (\cos\alpha + j\sin\alpha)^{-5}$

2 iii) cont

$$= \cos(-5\alpha) + j\sin(-5\alpha)$$

$$= \cos(-4.636476) + j\sin(-4.636476)$$

Adding 2π to angle

$$= \cos(1.646709) + j\sin(1.646709)$$

$$= -0.076 + 0.997j$$

2 iv)

$$(\sqrt{27} + 3j)^6$$

$$= (3\sqrt{3} + 3j)^6$$

$$= 3^6(\sqrt{3} + j)^6$$

$$= 3^6\left(2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)\right)^6$$

$$= 3^6 \times 2^6\left(\cos\frac{\pi}{6} + j\sin\frac{\pi}{6}\right)^6$$

$$= 46656(\cos\pi + j\sin\pi)$$

$$= -46,656$$

3) i)

$$(\cos(-\alpha) + j\sin(-\alpha))^8$$

$$= \cos(-8\alpha) + j\sin(-8\alpha)$$

$$= \cos 8\alpha - j\sin 8\alpha$$

3 ii)

$$\frac{(\cos\beta + j\sin\beta)^3}{(\cos\beta - j\sin\beta)^{-5}}$$

$$= \frac{\cos 3\beta + j\sin 3\beta}{(\cos(-\beta) + j\sin(-\beta))^{-5}}$$

$$= \frac{(\cos 3\beta + j\sin 3\beta)}{(\cos 5\beta + j\sin 5\beta)}$$

$$= \frac{(\cos\beta + j\sin\beta)^3}{(\cos\beta + j\sin\beta)^5}$$

$$= (\cos\beta + j\sin\beta)^{-2}$$

$$= \cos(-2\beta) + j\sin(-2\beta)$$

$$= \cos 2\beta - j\sin 2\beta$$

3 iii)

$$(\cos^2\gamma + j\sin\gamma\cos\gamma)^{10}$$

$$= (\cos\gamma(\cos\gamma + j\sin\gamma))^{10}$$

$$= \cos^{10}\gamma(\cos 10\gamma + j\sin 10\gamma)$$

3 iv)

$$(1 + \cos 2\delta + j\sin 2\delta)^{-4}$$

$$= (2\cos^2\delta + j\,2\sin\delta\cos\delta)^{-4}$$

$$= (2\cos\delta(\cos\delta + j\sin\delta))^{-4}$$

$$= \frac{1}{16\cos^4\delta}(\cos(-4\delta) + j\sin(-4\delta))$$

$$= \frac{\cos(4\delta) - j\sin(4\delta)}{16\cos^4\delta}$$

4 i)

$$(\cos\theta - j\sin\theta)^n$$

Let $\theta = -\phi$

$$= (\cos(-\phi) - j\sin(-\phi))^n$$

$$= (\cos\phi + j\sin\phi)^n$$

$$= \cos(n\phi) + j\sin(n\phi)$$

$$= \cos(-n\theta) + j\sin(-n\theta)$$

$$= \cos(n\theta) - j\sin(n\theta)$$

4ii) Let $z = \cos\theta + j\sin\theta$

then $\bar{z} = \cos\theta - j\sin\theta$

Now $z\bar{z} = \cos^2\theta + j\sin\theta\cos\theta$

$- j\sin\theta\cos\theta - j^2\sin^2\theta$

$= 1$

$\therefore (z\bar{z})^n = 1$

$\Rightarrow z^n\bar{z}^n = 1$

$\Rightarrow z^n = \dfrac{1}{\bar{z}^n}$

$\Rightarrow \bar{z}^n = \overline{(z^n)}$

$\Rightarrow \bar{z}^n = \overline{\cos n\theta + j\sin n\theta}$

$\Rightarrow \bar{z}^n = \cos n\theta - j\sin n\theta$

as required

//