1) $5x + 3y = 31$
$4x + 2y = 25$

$\begin{pmatrix} 5 & 3 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 31 \\ 25 \end{pmatrix}$

$\det M = 0$ ∴ Unique solution

$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 2 & -3 \\ -4 & 5 \end{pmatrix}\begin{pmatrix} 31 \\ 25 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} -13 \\ 1 \end{pmatrix} = \begin{pmatrix} 6\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$

$x = 6\frac{1}{2}, \quad y = -\frac{1}{2}$

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2) $3x + 9y = 12$
$2x + 6y = 15$

$\begin{pmatrix} 3 & 9 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 \\ 15 \end{pmatrix}$

$\det M = 0$

Inconsistent Parallel Lines

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3) $6x + 3y = 12$
$2x + y = 4$

Lines coincident ∴ infinite number of solutions given by

$x = \lambda, \quad y = 4 - 2\lambda$

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4) $6x - 3y = 11$
$y = 2x - 4$ ②

From $2x - y = 4$
②
Inconsistent Parallel lines

5) $x + y + z = 4$ ①
$2x + 3y - 4z = 3$ ②
$5x + 8y - 13z = 8$ ③

$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & -4 \\ 5 & 8 & -13 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 8 \end{pmatrix}$

$\det M = 1\begin{vmatrix} 3 & -4 \\ 8 & -13 \end{vmatrix} - 1\begin{vmatrix} 2 & -4 \\ 5 & -13 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix}$

$= -7 + 6 + 1 = 0$

② + 4①: $6x + 7y = 19$ ④

③ + 13①: $18x + 21y = 60$ ⑤

④ × 3: $18x + 21y = 57$ ⑥

⑤ and ⑥ inconsistent

No planes parallel ∴ planes form triangular prism

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6) $2x - y = 1$ ①
$3x + 2z = 13$ ②
$3y + 4z = 23$ ③

$\begin{pmatrix} 2 & -1 & 0 \\ 3 & 0 & 2 \\ 0 & 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 13 \\ 23 \end{pmatrix}$

$\det M = 2\begin{vmatrix} 0 & 2 \\ 3 & 4 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ 0 & 4 \end{vmatrix} + 0$

$= -12 + 12 = 0$

6 cont From ① $y = 2x - 1$

From ② $z = \frac{13 - 3x}{2}$

Subst in ③

$$3(2x-1) + 4\left(\frac{13-3x}{2}\right) = 23$$

$$6x - 3 + \frac{52 - 12x}{2} = 23$$

$$6x - 3 + 26 - 6x = 23$$

True for all x

so $x = \lambda$ say

$y = 2\lambda - 1$

$z = \frac{13 - 3\lambda}{2}$

Sheaf of planes

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7)

$$\begin{aligned} x + 2y + 4z &= 7 \\ 3x + 2y + 5z &= 21 \\ 4x + y + 2z &= 14 \end{aligned}$$

$$\begin{pmatrix} 1 & 2 & 4 \\ 3 & 2 & 5 \\ 4 & 1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 21 \\ 14 \end{pmatrix}$$

$$\det M = 1\begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} + 4\begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= -1 + 28 - 20 \neq 0$$

$\therefore$ unique solution

② − ① $2x + z = 14$ ④

2×③ − ② $5x - z = 7$ ⑤

④ + ⑤ $7x = 21$

$\Rightarrow x = 3$

$\Rightarrow z = 14 - 2\times 3 = 8$

Subst in ①

$3 + 2y + 32 = 7$

$2y = -28$

$y = -14$

$x = 3, \quad y = -14, \quad z = 8$

---

8)

$$\begin{aligned} 3x + 2y + z &= 2 \\ 5x + 3y - 4z &= 1 \\ x + y + 4z &= 5 \end{aligned}$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 5 & 3 & -4 \\ 1 & 1 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$\det M = 3\begin{vmatrix} 3 & -4 \\ 1 & 4 \end{vmatrix} - 2\begin{vmatrix} 5 & -4 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 48 - 48 + 2 \neq 0$$

$\therefore$ unique solution

② + ③ $6x + 4y = 6$ ④

② + 4① $17x + 11y = 9$ ⑤

④ × 11 $66x + 44y = 66$

⑤ × 4 $68x + 44y = 36$

$2x = -30$

$x = -15$

8 cont)

$$-90 + 4y = 6$$
$$4y = 96$$
$$y = 24$$

Subst in ①

$$-45 + 48 + z = 2$$
$$\Rightarrow z = -1$$

$x = -15$, $y = 24$, $z = -1$

②×3 − ④ $\quad 8x + 13z = 13$

③×3 − ① $\quad 16x + 26z = 30$

and not parallel

No solution ∴ planes form triangular prism

9)

$$2x + y - z = 5$$
$$8x + 4y - 4z = 20$$
$$-2x - y + z = -5$$

$$\begin{pmatrix} 2 & 1 & -1 \\ 8 & 4 & -4 \\ -2 & -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 20 \\ -5 \end{pmatrix}$$

All 3 planes coincident

Infinite number of solutions

Say $y = \lambda$, $z = \mu$

$$x = 5 - \lambda + \mu$$

10)

$$5x + 3y - 2z = 6$$
$$6x + 2y + 3z = 11$$
$$7x + y + 8z = 12$$

$$\begin{pmatrix} 5 & 3 & -2 \\ 6 & 2 & 3 \\ 7 & 1 & 8 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ 12 \end{pmatrix}$$

$$\det M = 5\begin{vmatrix} 2 & 3 \\ 1 & 8 \end{vmatrix} - 3\begin{vmatrix} 6 & 3 \\ 7 & 8 \end{vmatrix} - 2\begin{vmatrix} 6 & 2 \\ 7 & 1 \end{vmatrix}$$

$$= 65 - 81 + 16 = 0$$