

$$1 \text{ i)} \quad \underline{M} = \begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix}$$

$$\underline{M} - \lambda \underline{I} = \begin{pmatrix} 5-\lambda & 3 \\ 2 & 4-\lambda \end{pmatrix}$$

$$\text{Set } \det(\underline{M} - \lambda \underline{I}) = 0$$

$$(5-\lambda)(4-\lambda) - 6 = 0$$

$$20 - 4\lambda - 5\lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 9\lambda + 14 = 0$$

$$(\lambda - 7)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 7 \text{ or } \lambda = 2$$

$$\begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 7 \begin{pmatrix} x \\ y \end{pmatrix}$$

$$5x + 3y = 7x \quad \textcircled{1}$$

$$2x + 4y = 7y \quad \textcircled{2}$$

$$\text{From } \textcircled{1} \quad 3y = 2x$$

$$\text{From } \textcircled{2} \quad 3y = 2x$$

Let $y = 2h$, $\Rightarrow x = 3h$
For eigenvalue $\lambda = 7$

Eigen vector $\begin{pmatrix} 3h \\ 2h \end{pmatrix}$ or $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$
h a parameter.

$$\begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2 \begin{pmatrix} x \\ y \end{pmatrix}$$

$$5x + 3y = 2x \quad \textcircled{1}$$

$$2x + 4y = 2y \quad \textcircled{2}$$

$$\text{From } \textcircled{1} \quad 3y = -3x$$

$$\text{From } \textcircled{2} \quad 2y = -2x$$

$$\Rightarrow y = -x$$

For eigenvalue $\lambda = 2$

$$\text{eigen vector} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \text{ or } \begin{pmatrix} h \\ -h \end{pmatrix}$$

h a parameter

1ii)

$$\underline{M} = \begin{pmatrix} 7 & 2 \\ -12 & -4 \end{pmatrix}$$

$$\underline{M} - \lambda \underline{I} = \begin{pmatrix} 7-\lambda & 2 \\ -12 & -4-\lambda \end{pmatrix}$$

Set

$$\det(\underline{M} - \lambda \underline{I}) = 0$$

$$(7-\lambda)(-4-\lambda) + 24 = 0$$

$$-28 + 4\lambda - 7\lambda + \lambda^2 + 24 = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$(\lambda - 4)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = 4 \text{ or } \lambda = -1$$

When $\lambda = 4$

$$\begin{pmatrix} 7 & 2 \\ -12 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 4 \begin{pmatrix} x \\ y \end{pmatrix}$$

$$7x + 2y = 4x \quad \textcircled{1}$$

$$-12x - 4y = 4y \quad \textcircled{2}$$

$$\text{From } \textcircled{1} \quad 2y = -3x$$

$$\text{From } \textcircled{2} \quad 8y = -12x$$

Eigen vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ or $\begin{pmatrix} 2h \\ -3h \end{pmatrix}$

where h a parameter.

1ii) cont) When $\lambda = -1$

$$\begin{pmatrix} 7 & 2 \\ -12 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -1 \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{aligned} 7x + 2y &= -x & \textcircled{1} \\ -12x - 4y &= -y & \textcircled{2} \end{aligned}$$

From $\textcircled{1}$ $2y = -8x$

From $\textcircled{2}$ $3y = -12x$

Eigenvector $\begin{pmatrix} h \\ -4h \end{pmatrix}$ or $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

h a parameter,

2i)

$$M = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

$$M - \lambda I = \begin{pmatrix} 3-\lambda & 0 & 0 \\ 0 & 2-\lambda & 1 \\ 0 & 0 & -1-\lambda \end{pmatrix}$$

$$\det M - \lambda I$$

$$= (3-\lambda)[(2-\lambda)(-1-\lambda)]$$

$$= (3-\lambda)(-2+\lambda-2\lambda+\lambda^2)$$

$$= (3-\lambda)(\lambda^2 - \lambda - 2)$$

$$= 3\lambda^2 - \lambda^3 - 3\lambda + \lambda^2 - 6 + 2\lambda$$

$$= -\lambda^3 + 4\lambda^2 - \lambda - 6$$

Set $\det(M - \lambda I) = 0$

$$\Rightarrow \lambda^3 - 4\lambda^2 + \lambda + 6 = 0$$

$\lambda = -1$ a solution

$$\begin{array}{r} \lambda^2 - 5\lambda + 6 \\ (\lambda+1) \overline{) \lambda^3 - 4\lambda^2 + \lambda + 6} \\ \lambda^3 + \lambda^2 \\ \hline -5\lambda^2 + \lambda \\ -5\lambda^2 - 5\lambda \\ \hline 6\lambda + 6 \\ 6\lambda + 6 \\ \hline \end{array}$$

$$(\lambda+1)(\lambda^2 - 5\lambda + 6) = 0$$

$$(\lambda+1)(\lambda-3)(\lambda-2) = 0$$

Solution $\lambda = -1, 3$ or 2

When $\lambda = -1$

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -1 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{aligned} 3x &= -x & \textcircled{1} \\ 2y + z &= -y & \textcircled{2} \\ -z &= -z & \textcircled{3} \end{aligned}$$

From $\textcircled{1}$ $x = 0$

From $\textcircled{2}$ $z = -3y$

Eigenvector = $k \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}$

When $\lambda = 3$

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{aligned} 3x &= 3x & \textcircled{1} \\ 2y + z &= 3y & \textcircled{2} \\ -z &= 3z & \textcircled{3} \end{aligned}$$

From $\textcircled{3}$ $z = 0$ Eigenvector $k \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
 From $\textcircled{2}$ $y = 0$

2 i) When $\lambda = 2$
cont)

$$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2 \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{aligned} 3x &= 2x & \textcircled{1} \\ 2y + z &= 2y & \textcircled{2} \\ -z &= 2z & \textcircled{3} \end{aligned}$$

From $\textcircled{3}$ $z = 0$ eigenvect $\propto \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$
From $\textcircled{1}$ $x = 0$