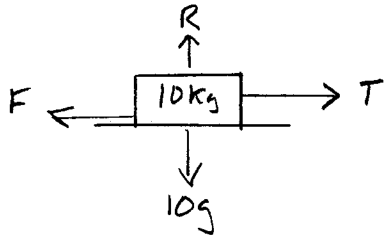
1)



i) On point of sliding $F = \mu R$

$F = \mu \times 10g$

Also $T = F$

$\therefore \quad 9.8 = 98\mu$

$\Rightarrow \mu = 0.1$

ii)

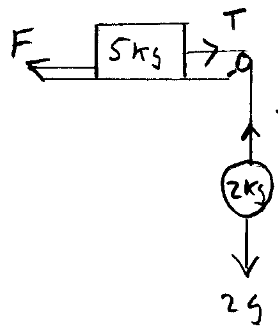
If $T = 49\text{ N}$

$49 = 98\mu$

$\Rightarrow \mu = 0.5$

2)

i)



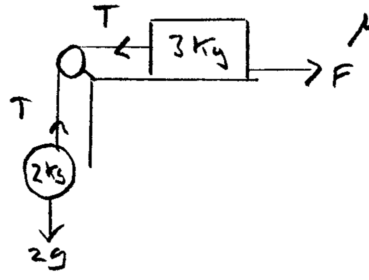
$\mu = \frac{1}{2}$

Max $F = \mu R = \frac{1}{2} \times 5g$

$= 2.5g$

$\therefore$ block does not move since $T = 2g$. Actual $F = 2g$

ii)



$\mu = \frac{1}{4}$

In equilibrium $T = 2g$

Max $F = \mu R = \frac{1}{4} \times 3g = \frac{3}{4}g$

$\therefore$ not in equilibrium horizontally

Eqns of motion

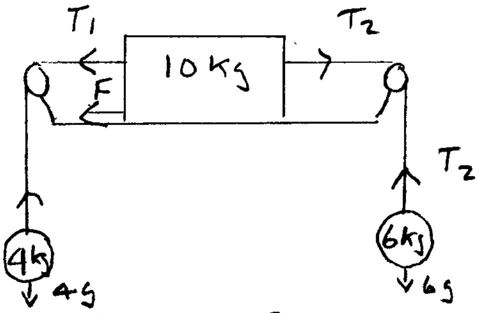
For sphere $2g - T = 2a$

For block $T - \frac{3}{4}g = 3a$

Add $\frac{5}{4}g = 5a$

$\Rightarrow a = \frac{1}{4}g = 2.45\text{ ms}^{-2}$

iii)



$\mu = 0.3$

In equilibrium $T_2 = T_1 + F$

$6g = 4g + F$

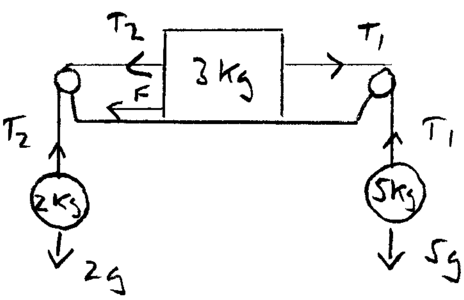
$\Rightarrow F = 2g$

Max $F = \mu R = 0.3 \times 10g = 3g$

$\therefore$ block does not move

Actual $F = 2g$

iv)



$\mu = \frac{1}{4}$

2iv) cont) In equilibrium

$$T_1 = T_2 + F$$

$$5g = 2g + F$$

$$\Rightarrow F = 3g$$

$$\text{Max } F = \mu R = \tfrac{1}{4} \times 3g$$

$$= \tfrac{3}{4} g$$

$\therefore$ not in equilibrium

Eqns of motion

For 5kg sphere $\quad 5g - T_1 = 5a$

For 2kg sphere $\quad T_2 - 2g = 2a$

For Block $\quad T_1 - T_2 - \tfrac{3}{4}g = 3a$

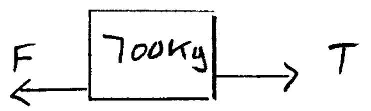
Add $\quad \tfrac{9}{4} g = 10a$

$$\Rightarrow a = \frac{\tfrac{9}{4} \times 9.8}{10}$$

$$a = 2.205 \text{ ms}^{-2}$$

3)

$\mu = 0.7$



$$\text{Max } F = \mu R = 0.7 \times 700g$$

To bring caravan to the point of moving

$$T = 0.7 \times 700g$$

$$T = 4802 \text{ N}$$

4)i) $u = 10 \text{ ms}^{-1}$ $\quad s = 49\text{m}$

Using $\quad v^2 = u^2 + 2as$

At rest $\quad 0^2 = 10^2 + 2a \times 49$

$$-100 = 98a$$

$$a = -\frac{100}{98} \text{ ms}^{-2}$$

$$a = -1.02 \text{ ms}^{-2}$$

4 ii)

Friction provides force for deceleration

$$F = ma$$

$$F = 0.1 \times -1.02$$

$$F = -0.102 \text{ N}$$

Magnitude of friction $= 0.102$ N

4 iii) Max F when moving so

$$F = \mu R$$

$$0.102 = 0.1g \times \mu$$

$$\mu = \frac{0.102}{0.1 \times 9.8}$$

$$\mu = 0.104$$

4 iv)

Same distance since twice the mass gives twice the frictional force so deceleration is the same. Distance $= 49$m

5) $u = 12\,ms^{-1}$, $s = 9\,m$

Using $v^2 = u^2 + 2as$

$0 = 12^2 + 18a$

$\Rightarrow a = -\dfrac{144}{18} = -8\,ms^{-2}$

Friction causes deceleration

$F = ma = -8m$

Magnitude of friction $= 8m$

But $F = \mu R$ when moving

$\therefore 8m = \mu mg$

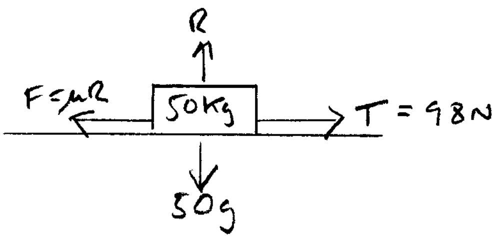
$\Rightarrow \mu = \dfrac{8m}{mg} = \dfrac{8}{g}$

$\mu = 0.816$ to 3 s.f.

6)

i) Smoother contact with floor

ii)



At constant velocity $T = F$

$\Rightarrow \mu R = 98$

$\mu \times 50g = 98$

$\mu = \dfrac{98}{50 \times 9.8}$

$\mu = 0.2$

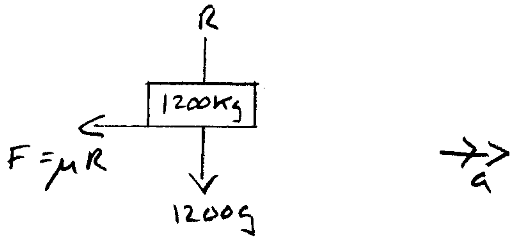
6iii) Again $T = F$ at const. vel.

$T = \mu R = 0.2 \times 70g$

$T = 137.2\,N$

7)

i)



N2L $F = ma$

$-\mu R = ma$

$-0.75 \times 1200g = 1200a$

$-0.75g = a$

$a = -7.35\,ms^{-2}$

Deceleration $= 7.35\,ms^{-2}$

7ii) $v^2 = u^2 + 2as$

$v^2 = 30^2 + 2 \times (-7.35) \times 40$

$v^2 = 312$

$v = 17.7\,ms^{-1}$ to 3 s.f.

7iii)

For next part of journey

$a = -0.8g = -7.84\,ms^{-2}$

Using $v^2 = u^2 + 2as$

At rest $v = 0$

$0^2 = 312 - 2 \times 7.84s$

$15.68s = 312$

7iii) cont

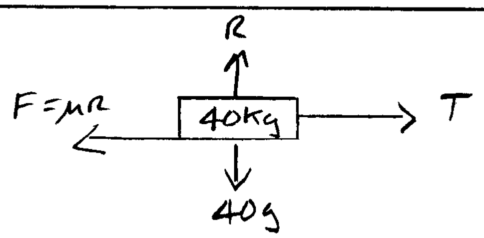
$s = 19.9\text{ m}$ to 3 s.f

Total distance after braking

$= 40 + 19.9 = 59.9\text{ m}$

8)

i)



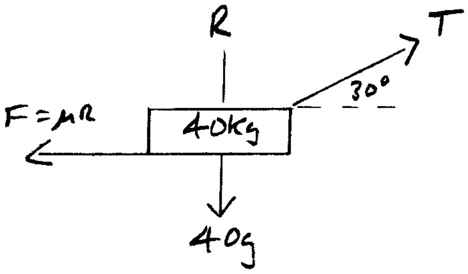
At constant velocity

$T = F = \mu R$

$T = 0.15 \times 40g$

$T = 58.8\text{ N}$

8ii)



Horizontally

$T\cos 30^\circ = \mu R$

① $T\cos 30 = 0.15R$

Vertically

② $R + T\sin 30 = 40g$

From ① $R = \dfrac{T\cos 30^\circ}{0.15}$

Subst in ② $\dfrac{T\cos 30^\circ}{0.15} + T\sin 30^\circ = 40g$

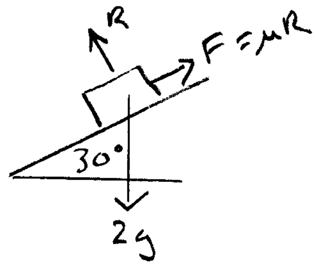
$T\left(\dfrac{\cos 30^\circ}{0.15} + \sin 30^\circ\right) = 40 \times 9.8$

$T = \dfrac{40 \times 9.8}{\left(\dfrac{\cos 30^\circ}{0.15} + \sin 30^\circ\right)}$

$T = 62.5\text{ N}$

9)

i)



$R = 2g\cos 30^\circ$

Parallel to slope

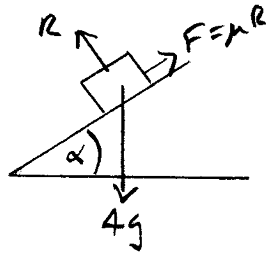
$F = \mu R = 2g\sin 30^\circ$

$\mu \times 2g\cos 30^\circ = 2g\sin 30^\circ$

$\mu = \dfrac{2g\sin 30^\circ}{2g\cos 30^\circ} = \tan 30^\circ$

$\mu = \dfrac{1}{\sqrt{3}} = 0.577$ to 3 s.f.

ii)



$\mu = 0.7$

$R = 4g\cos\alpha$

Parallel to slope

$F = \mu R = 4g\sin\alpha$

$\mu \times 4g\cos\alpha = 4g\sin\alpha$

$\mu = \tan\alpha$

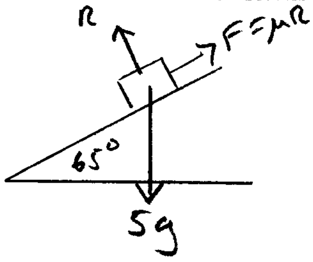
9ii) (cont)

$$0.7 = \tan\alpha$$

$$\alpha = \tan^{-1} 0.7 = 35.0^\circ$$

to 3s.f.

9iii)



$$R = 5g\cos 65^\circ$$

Parallel to slope

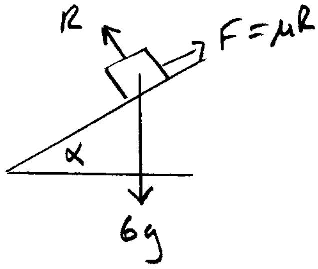
$$F = \mu R = 5g\sin 65^\circ$$

$$\mu \times 5g\cos 65^\circ = 5g\sin 65^\circ$$

$$\mu = \tan 65^\circ$$

$$\mu = 2.14 \quad \text{to 3 s.f.}$$

9iv)



$$R = 6g\cos\alpha$$

Parallel to slope

$$F = \mu R = 6g\sin\alpha$$

$$\mu \times 6g\cos\alpha = 6g\sin\alpha$$

$$\mu = \tan\alpha$$

$$1.2 = \tan\alpha$$

$$\alpha = \tan^{-1} 1.2 = 50.2^\circ$$

to 3s.f.

Note

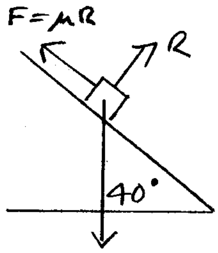
Because the brick was on the point of slipping throughout question 9, we could just use

$$\mu = \tan\alpha$$

to relate μ to the angle of slope. Notice that this relationship does not depend on the mass

10)

i)



$$\mu = 0.25$$

ii)

For slope angle θ

$$R = mg\cos\theta$$

Parallel to slope $F = ma$ (N2L)

$$mg\sin\theta - \mu R = ma$$

$$mg\sin\theta - \mu \times mg\cos\theta = ma$$

$$g\sin 40^\circ - 0.25g\cos 40^\circ = a$$

$$a = 4.4225 \text{ ms}^{-2}$$

$$a = 4.42 \text{ ms}^{-2} \quad \text{to 3 s.f.}$$

iii)

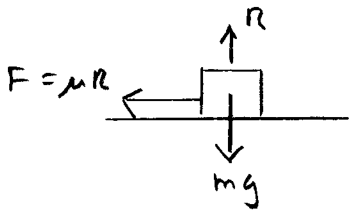
Using $v^2 = u^2 + 2as$

$$v^2 = 0^2 + 2 \times 4.4225 \times 3$$

$$\Rightarrow v = 5.1512 \text{ ms}^{-1}$$

$$v = 5.15 \text{ ms}^{-1} \quad \text{to 3 s.f.}$$

10iv)



Frictional force on horizontal section

$= -\mu m g$

N2L $\quad F = ma$

$-\mu m g = ma$

$\Rightarrow \quad a = -\mu g$

$a = -0.25 \times 9.8$

$a = -2.45 \text{ ms}^{-2}$

Using $\quad v^2 = u^2 + 2as$

At rest $v = 0$

$0^2 = 5.1512^2 - 2 \times 2.45 s$

$4.9 s = 5.1512^2$

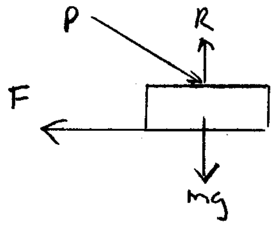
$s = \dfrac{5.1512^2}{4.9}$

$s = 5.415 \text{ m}$

Horizontal section $= 5.42$ m

to 3 s.f.

11)
i)

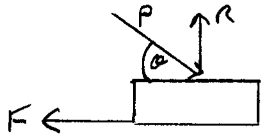


$\mu = 0.3$

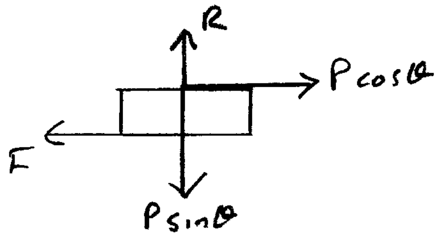
Direction of movement

P is force exerted by pusher

ii)



Resolving P vertically and horizontally



$R = P \sin\theta$

At constant velocity $\quad F = P\cos\theta$

$\Rightarrow \quad \mu R = P\cos\theta$

$\Rightarrow \quad \mu P \sin\theta = P\cos\theta$

$\Rightarrow \quad \dfrac{\mu P \sin\theta}{P\cos\theta} = 1$

$\Rightarrow \quad \mu \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \dfrac{1}{\mu} = \dfrac{1}{0.3}$

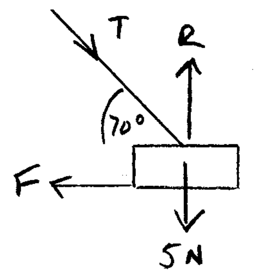
$\theta = \tan^{-1}\left(\dfrac{1}{0.3}\right)$

$\theta = 73.3^\circ$

Angle independent of pushing force as multiplying the pushing force by a scale factor would multiply the forward force, the reaction and therefore the friction by the same scale factor.

Thus horizontal equilibrium would be maintained and the mop would continue with constant velocity.

11 iii)



$\mu = 0.3$

At constant velocity

$$T\cos 70^\circ = F$$

$$T\cos 70^\circ = \mu R$$

$$T\cos 70^\circ = \mu(5 + T\sin 70^\circ)$$

$$T\cos 70^\circ = 0.3(5 + T\sin 70^\circ)$$

$$T\cos 70^\circ = 1.5 + 0.3T\sin 70^\circ$$

$$T\cos 70^\circ - 0.3T\sin 70^\circ = 1.5$$

$$T(\cos 70^\circ - 0.3\sin 70^\circ) = 1.5$$

$$T = \frac{1.5}{(\cos 70^\circ - 0.3\sin 70^\circ)}$$

$$T = 25.0 \text{ N} \quad \text{to 3 s.f.}$$

11 iv)

Model not applicable since wheels do not slip

12)

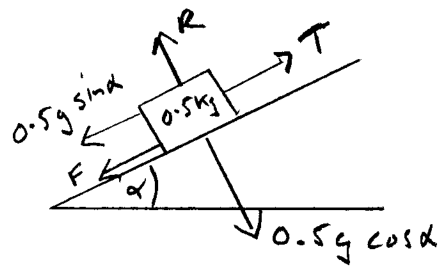
Constant velocity when $\tan\alpha = \mu = 0.3$

$$\alpha = \tan^{-1} 0.3$$

$$\alpha = 16.7^\circ$$

Top part $\alpha > 16.7^\circ$
Middle part $\alpha = 16.7^\circ$
Bottom part $\alpha < 16.7^\circ$

13)



On point of sliding

$$T = 0.5g\sin\alpha + F$$

$$T = 0.5g\sin\alpha + \mu R$$

$$T = 0.5g\sin\alpha + 0.7 \times 0.5g\cos\alpha$$

If $\sin\alpha = 0.6$, $\cos\alpha = \sqrt{1 - 0.6^2} = 0.8$

$$T = 0.5 \times 9.8 \times 0.6 + 0.7 \times 0.5 \times 9.8 \times 0.8$$

$$T = 5.684 \text{ N}$$

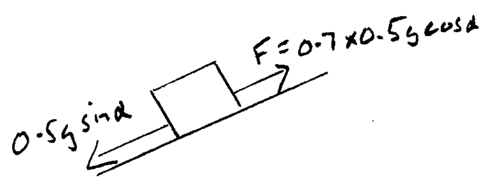
$$T = 5.68 \text{ N} \quad \text{to 3 s.f.}$$

ii)

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{0.6}{0.8} = 0.75$$

since $\tan\alpha > \mu$

and string is slack, block would begin to accelerate down slope.



Resultant force down slope
$= 0.5 \times 9.8 \times 0.6 - 0.7 \times 0.5 \times 9.8 \times 0.8$

$= 0.196$ N

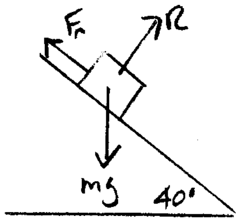
Will slow down once $T > 0.196$ N again

14) i) At constant speed $\tan\alpha = \mu$

$\tan 11° = \mu$

$\mu = 0.194$ to 3 s.f.

14 ii)



$\mu = 0.19438$

Parallel to slope $F = ma$

$mg\sin 40° - F_r = ma$

$mg\sin 40° - \mu R = ma$

$mg\sin 40° - 0.19438\,mg\cos 40° = ma$

$g(\sin 40° - 0.19438\cos 40°) = a$

$a = 4.84 \text{ ms}^{-2}$

14 iii)

Speed at end of chute = speed at bottom of steep part since swimmer then travels with constant velocity

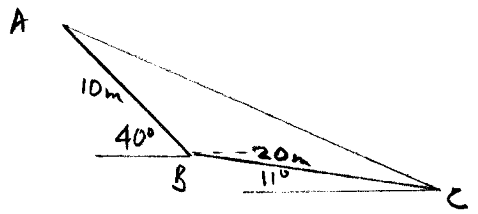
$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 4.84 \times 10$

$v^2 = 96.8$

$v = 9.84 \text{ ms}^{-1}$

14 iv)



$\angle ABC = (180 - 40) + 11 = 151°$

Cosine rule

$AC^2 = 10^2 + 20^2 - 2 \times 10 \times 20\cos 151°$

$\Rightarrow AC = 29.152 \text{ m}$

Sine rule to find $\angle BCA$

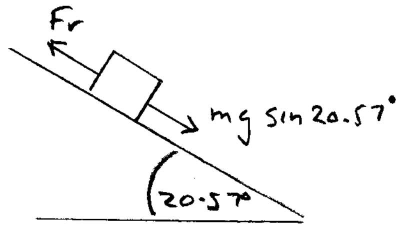
$$\frac{10}{\sin\angle BCA} = \frac{29.152}{\sin 151}$$

$$\sin(\angle BCA) = \frac{\sin 151°}{29.152} \times 10$$

$\Rightarrow \angle BCA = 9.57°$

Slope of $AC = 9.57 + 11°$

$= 20.57°$



$F = ma$

$mg\sin 20.57° - \mu mg\cos 20.57°$

$= ma$

$g(\sin 20.57 - 0.19438\cos 20.57°)$

$= a$

$a = 1.66 \text{ ms}^{-2}$

Slope is 29.152 m long so using

$v^2 = u^2 + 2as$

14 iv) cont

$$v^2 = 0^2 + 2 \times 1.66 \times 29.152$$

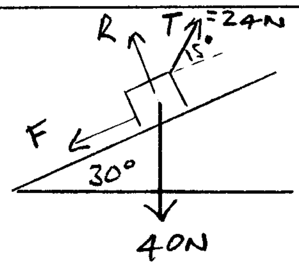
$$\Rightarrow v = 9.84 \text{ m s}^{-1}$$

This is the same speed as for the previous design.

You will see later in the course that this problem could have been solved much more easily using a work-energy approach.

Basically, the gain in kinetic energy is equal to the loss in potential energy − the work done by the frictional force. This is the same if the start and end points are the same.

15) i)



Steady speed so

$$T\cos 15^\circ = F + 40\sin 30^\circ$$

$$24\cos 15^\circ - 40\sin 30^\circ = F$$

$$\underline{F = 3.18 \text{ N}} \quad \text{to 3 s.f.}$$

Perpendicular to slope

$$R + T\sin 15 = 40\cos 30^\circ$$

$$R = 40\cos 30^\circ - 24\sin 15^\circ$$

$$R = 28.4 \text{ N} \quad \text{to 3 s.f.}$$

15 ii)

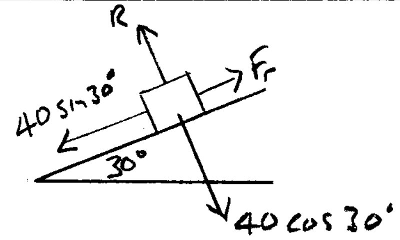
$$F = \mu R$$

$$\therefore \mu = \frac{F}{R} = \frac{3.18}{28.4}$$

$$\mu = 0.112$$

$\therefore \mu$ is slightly more than 0.1

15 iii)



$\mu = 0.1$

Parallel to slope $F = ma$ N2L

$$40\sin 30 - F_r = ma$$

$$40\sin 30^\circ - \mu R = \frac{40}{9.8}a$$

$$\frac{40\sin 30^\circ - 0.1 \times 40\cos 30^\circ}{\frac{40}{9.8}} = a$$

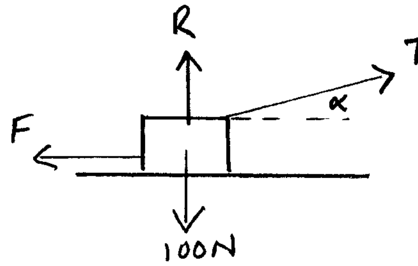
$$\underline{a = 4.05 \text{ m s}^{-2}} \quad \text{to 3 s.f.}$$

$$F_r = \mu R$$

$$F_r = 0.1 \times 40\cos 30^\circ$$

$$F_r = 3.46 \text{ N} \quad \text{to 3 s.f.}$$

16)



$\mu = 0.4$

16i) a) At steady speed

$$T\cos\alpha = F$$

$$T\cos\alpha = \mu R$$

$$T\cos\alpha = \mu(100 - T\sin\alpha)$$

$$T\cos\alpha = \mu \times 100 - \mu T\sin\alpha$$

$$T\cos\alpha + \mu T\sin\alpha = 100\mu$$

$$T(\cos\alpha + \mu\sin\alpha) = 100\mu$$

$$T = \frac{100\mu}{\cos\alpha + \mu\sin\alpha}$$

When $\alpha = 10°$, $\mu = 0.4$

$$T = \frac{100 \times 0.4}{\cos 10° + 0.4\sin 10°}$$

$\underline{T = 37.9 \text{ N}}$ to 3 s.f.

b) When $\alpha = 20°$

$$T = \frac{100 \times 0.4}{\cos 20° + 0.4\sin 20°}$$

$\underline{T = 37.2 \text{ N}}$ to 3 s.f

c) When $\alpha = 30°$

$$T = \frac{100 \times 0.4}{\cos 30° + 0.4\sin 30°}$$

$\underline{T = 37.5 \text{ N}}$ to 3 s.f.

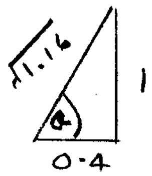
16ii)

$$T = \frac{40}{\cos\alpha + 0.4\sin\alpha}$$

16 iii) This part depends on a technique from Core 4 known as $\sin(\theta + \alpha)$

$$\cos\alpha + 0.4\sin\alpha$$

$$= R\sin(\alpha + \theta)$$

where 

$\tan\theta = \left(\frac{1}{0.4}\right)$

$R = \sqrt{1.16}$

$$\theta = \tan^{-1}\left(\frac{1}{0.4}\right) = 68.2°$$

$\therefore \cos\alpha + 0.4\sin\alpha$

$$= \sqrt{1.16}\sin(\alpha + 68.2°)$$

$$\therefore T = \frac{40}{\sqrt{1.16}\sin(\alpha + 68.2°)}$$

Min value when denominator takes greatest value

ie when $\alpha + 68.2° = 90°$

$\therefore \alpha = 21.8°$

for T to be a minimum