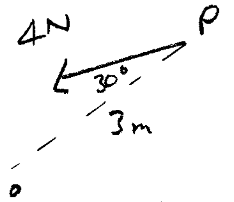
MEI MECHANICS 2 MOMENTS EXERCISE 3B

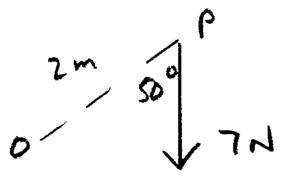
1) i)



Moment = $F \times d\sin\theta$

$= 4 \times 3\sin 30 = 6$ Nm

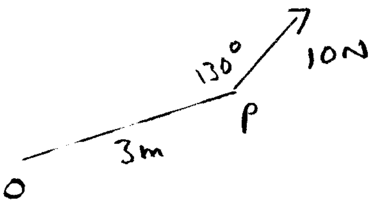
ii)



Moment $= -7 \times 2\sin 50°$

$= -10.7$ Nm to 3 s.f.

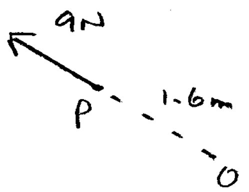
iii)



Moment $= 10 \times 3\sin 130°$

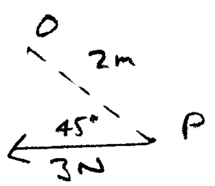
$= 23.0$ Nm to 3 s.f.

iv)



Moment = 0

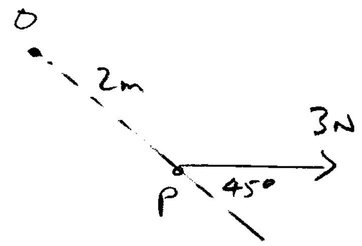
v)



Moment $= -3 \times 2\sin 45$

$= -4.24$ Nm

vi)



Moment $= 3 \times 2\sin 45°$

$= 4.24$ Nm

2) Total moment =

$65R\sin 70° + 51R - 115R\sin 77$

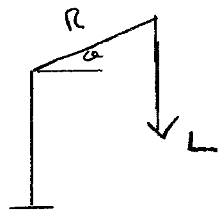
$= 0.027R$ Nm

where R is radius of roundabout

Moment is > 0 ie anti-clockwise

so David and Hannah win

3)



Ignore moment of weight of arm as not mentioned in question.

Maximum allowed moment when $\theta = 25°$, $L = 5000g$

$= R \times 5000g\cos 25$

(All moments of loads are clockwise so can neglect signs)

i) When $\theta = 40°$, max safe load L given by

$R \times Lg\cos 40 = R \times 5000g\cos 25°$

MEI MECHANICS 2 MOMENTS EXERCISE 3B

3i) (cont) $\Rightarrow L = \dfrac{R \times 5000g \cos 25^\circ}{Rg \cos 40^\circ}$

$$L = \frac{5000 \cos 25}{\cos 40^\circ}$$

$$L = 5916 \text{ kg}$$

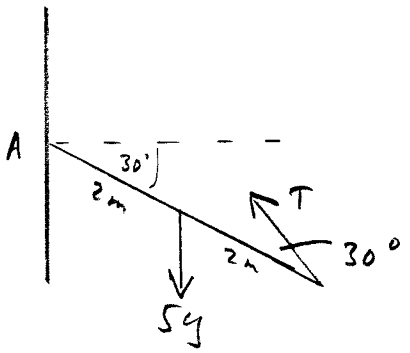
n'est kg

3ii) By a similar arguement maximum safe load at angle θ given by

$$L = \frac{5000 \cos 25^\circ}{\cos \theta}$$

4)

i)



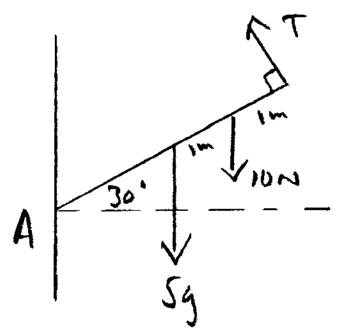
Taking moments about A

$$T \times 4 \sin 30^\circ = 5g \times 2 \cos 30^\circ$$

$$T = \frac{10g \cos 30^\circ}{4 \sin 30^\circ}$$

$$T = 42.4 \text{ N} \quad \text{to 3 s.f.}$$

4ii)



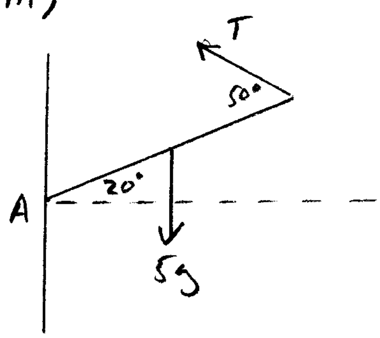
Taking moments about A

$$5g \times 2\cos 30 + 10 \times 3 \cos 30 = T \times 4$$

$$T = \frac{5 \times 9.8 \times 2\cos 30 + 30 \cos 30}{4}$$

$$T = 27.7 \text{ N}$$

4iii)



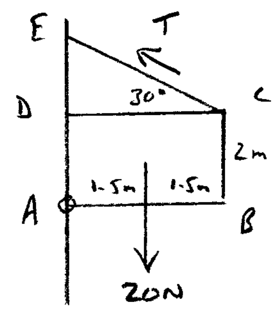
Taking moments about A

$$5g \times 2 \cos 20^\circ = T \times 4 \sin 50^\circ$$

$$T = \frac{10g \cos 20^\circ}{4 \sin 50^\circ}$$

$$T = 30.1 \text{ N}$$

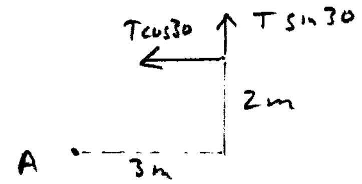
5)



i) Vertically $T \sin 30^\circ$

Horizontally $T \cos 30^\circ$

ii)



Total moment of T about A

$= T \sin 30 \times 3 + T \cos 30 \times 2 \quad \text{Nm}$

$= 3T \sin 30^\circ + 2T \cos 30^\circ \quad \text{NM}$

5iii) $20 \times 1.5 = 30 \text{ Nm}$

5iv)

In equilibrium so

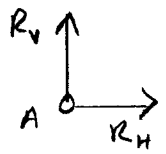
$$30 \text{ Nm} = 3T\sin 30^\circ + 2T\cos 30^\circ$$

$$30 = T(3\sin 30^\circ + 2\cos 30^\circ)$$

$$T = \frac{30}{3\sin 30 + 2\cos 30^\circ}$$

$$T = 9.28 \text{ N} \quad \text{to 3 s.f.}$$

5v)



Horizontally $R_H = T\cos 30$

$$R_H = 9.282 \times \cos 30^\circ$$

$$R_H = 8.04 \text{ N} \quad \text{to 2dp}$$

Vertically

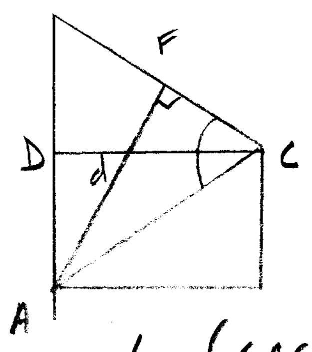
$$R_V + T\sin 30 = 20$$

$$R_V = 20 - 9.282\sin 30$$

$$R_V = 15.36 \text{ N} \quad \text{to 2.dp}$$

5vi)

a)



$$\tan(\angle ACD) = \frac{2}{3}$$

$$\Rightarrow \angle ACD = 33.7^\circ$$

b)

$$\therefore \angle ACF = 33.7 + 30 = 63.7^\circ$$

$$AC^2 = 2^2 + 3^2$$

$$\Rightarrow AC = \sqrt{13}$$

$$\sin 63.7 = \frac{d}{\sqrt{13}}$$

$$\Rightarrow d = \sqrt{13}\sin 63.7^\circ$$

$$d = 3.232 \text{ m}$$

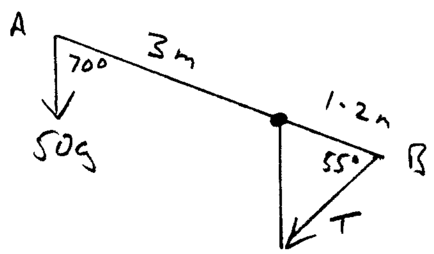
vii)

In equilibrium so

$$T \times 3.232 = 30$$

$$T = \frac{30}{3.232} = 9.28 \text{ N}$$

6)



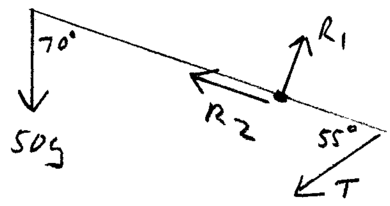
i) Taking moments about pivot

$$50g \times 3\sin 70 = T \times 1.2\sin 55^\circ$$

$$T = \frac{50 \times 9.8 \times 3\sin 70^\circ}{1.2\sin 55^\circ}$$

$$T = 1405 \text{ N}$$

6ii)



Parallel to jib

$T\cos 55° + R_2 = 50g\cos 70$

$R_2 = 50g\cos 70° - 1405\cos 55°$

$R_2 = -638$ N

Perpendicular to jib

$R_1 = 50g\sin 70 + T\sin 55°$

$R_1 = 50 \times 9.8\sin 70 + 1405\sin 55$

$R_1 = 1611$ N

$R_2 = -638$ N indicates it has magnitude 638N and is in opposite direction to that shown on diagram.

6iii) Moment about A

$= R_1 \times 3 - T\sin 55° \times 4.2$

$= 1611 \times 3 - 1405\sin 55° \times 4.2$

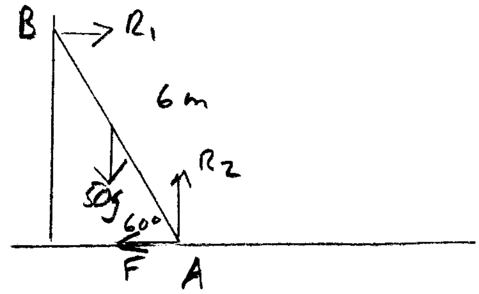
$= -0.816$ Nm ≈ 0

(due to rounding)

6iv)
a) If we continue to neglect the weight of the jib (somewhat unrealistic) T will reduce to 0 and the jib would not move.

b) If rope with tension T snaps, the moment of the 50g weight is unopposed and A would rotate anti-clockwise about the pivot.

7)
i)



ii) 0 since plank in equilibrium

iii) Taking moments about A

$R_1 \times 6\sin 60° = 50g \times 3\cos 60°$

$\Rightarrow R_1 = \dfrac{50g \times 3\cos 60°}{6\sin 60°}$

$R_1 = 141.45$ N

$R_1 = 141$ N to 3 s.f.

iv) $F = R_1 = 141$ N

$R_2 = 50g$

$F \leq \mu R_2$

$141.45 \leq \mu 490$

$\dfrac{141.45}{490} \leq \mu$

$\mu \geq 0.289$ to 3 s.f.

7v) Total Moment about B

$= R_2 \times 6\cos 60 - F \times 6\sin 60 - 50g \times 3\cos 60^\circ$

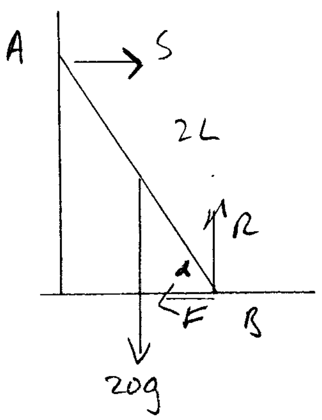
$= 50g \times 6\cos 60 - 141.45 \times 6\sin 60^\circ - 50g \times 3\cos 60^\circ$

$= 0.004$ Nm

≈ 0 (due to rounding)

8)

i)



a) ii) $\alpha = 60^\circ$

Moments about B

$S \times 2L\sin 60 = 20g \times L\cos 60$

$S = \dfrac{20g \times \cancel{L}\cos 60}{2\cancel{L}\sin 60}$

$S = 56.6$ N

Resolving horizontally

$F = 56.6$ N also

Resolving vertically

$R = 20g = 196$ N

iii) Since $F \le \mu R$

$56.5803 \le \mu \times 196$

$\mu \geq 0.289$

b) ii)

$S = \dfrac{20g \times \cancel{L}\cos 45}{2\cancel{L}\sin 45^\circ}$

$S = 98$ N

Resolving horizontally

$F = 98$ N

Resolving vertically

$R = 20g = 196$ N

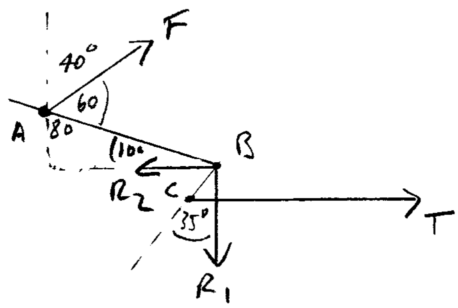
iii) Since $F \le \mu R$

$98 \le \mu \times 196$

$\mu \geq \frac{1}{2}$

9)

i)



ii) $F\sin 60 \times 0.350 = T \times 0.060\cos 35^\circ$

$F = \dfrac{1000 \times 0.060\cos 35^\circ}{0.350\sin 60^\circ}$

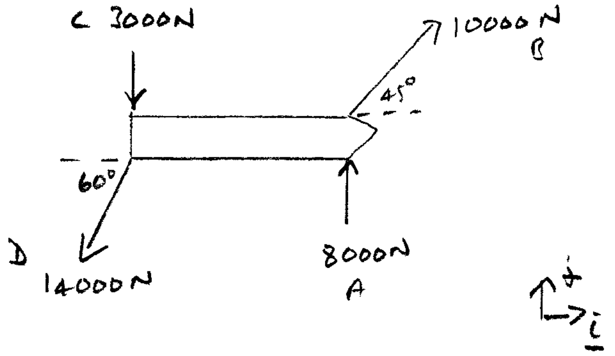
$F = 162.15$ N $= 162$ N to 3s.f.

9iii) $F \sin 60 \times 0.350 = T \times 0.060 \cos 35°$

$$\frac{10 \sin 60 \times 0.350}{0.060 \cos 35°} = T$$

$$T = 61.7 \text{ N}$$

10) i)



Resultant force on ship

$$\begin{pmatrix} 0 \\ 8000 \end{pmatrix} + \begin{pmatrix} 10000 \cos 45 \\ 10000 \sin 45 \end{pmatrix} + \begin{pmatrix} 0 \\ -3000 \end{pmatrix} + \begin{pmatrix} -14000 \cos 60 \\ -14000 \sin 60 \end{pmatrix}$$

$$= \begin{pmatrix} 71.0678 \\ -53.2878 \end{pmatrix} \text{ N}$$

Magnitude of resultant

$$= \sqrt{71.0678^2 + (-53.2878)^2}$$

$$= 88.8 \text{ N}$$

$$\therefore < 100 \text{ N}$$

10 ii) Total moment =

$8000 \times 50 + 10000 \sin 45 \times 50$

$- 10000 \cos 45 \times 15 + 3000 \times 70$

$+ 14000 \sin 60 \times 70 - 14000 \cos 60 \times 15$

$= 1{,}601{,}192$ Nm

$= 1{,}600{,}000$ Nm to 3 s.f.

10 iii) Before wind

$$F_A = 8000, \quad F_C = 3000$$

$$F_A - F_C = 5000$$

If wind provides 2000N from South

$$F_A - F_C = 3000 \quad (1)$$

to ensure no sideways force.

Moment has to be unchanged so

$$8000 \times 50 + 3000 \times 70 = 50 F_A + 70 F_C$$

$$\Rightarrow 50 F_A + 70 F_C = 610000$$

$$5 F_A + 7 F_C = 61000 \quad (2)$$

From (1) $F_A = 3000 + F_C$

Subst in (2)

$$5(F_C + 3000) + 7F_C = 61000$$

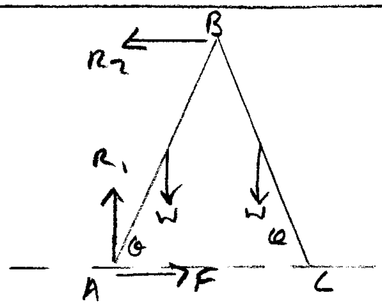
$$5F_C + 15000 + 7F_C = 61000$$

$$12 F_C = 46000$$

$$F_C = 3833 \text{ N}$$

$$\Rightarrow F_A = 6833 \text{ N}$$

11)



i) Force on each rod at B is horizontal, because they oppose each other and due to symmetry any vertical components would be identical, ∴ vertical components must be zero.

ii) See diagram above

Vertical equilibrium

$\Rightarrow R_1 = W$

Horizontal equilibrium

$\Rightarrow R_2 = F$

On point of slipping so

$F = \mu R_1 = \mu W$

iv)

Moments about A

Let rod be 2L in length

$W \times L\cos\theta = R_2 \times 2L\sin\theta$

$W \times \cos\theta = F \times 2\sin\theta$

$W\cos\theta = \mu W \times 2\sin\theta$

$\cos\theta = 2\mu\sin\theta$

$\frac{1}{2\mu} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

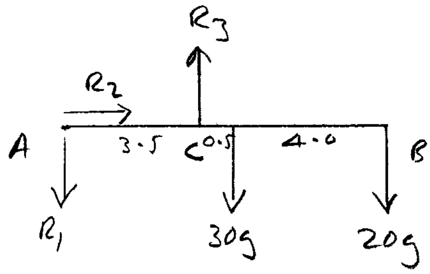
v) As $\mu \to 0$ $\frac{1}{2\mu} \to \infty$

$\Rightarrow \theta \to 90^\circ$

Lower μ requires rods to be nearer to vertical

12)

i)



ii) Moments about A

$R_3 \times 3.5 = 30g \times 4 + 20g \times 8$

$3.5R_3 = 280g$

$R_3 = 80g = 784$ N

(vertically upwards)

iii)

Resolving horizontally

$\Rightarrow R_2 = 0$

since no other horizontal forces

Resolving vertically

$R_1 + 30g + 20g = 80g$

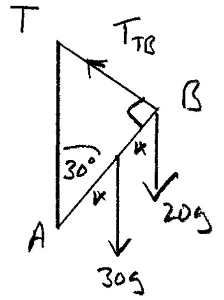
$\Rightarrow R_1 = 30g = 294$ N

(vertically downwards)

∴ resultant reaction at A

= 294 N vertically downwards

12iv)



Moments about A

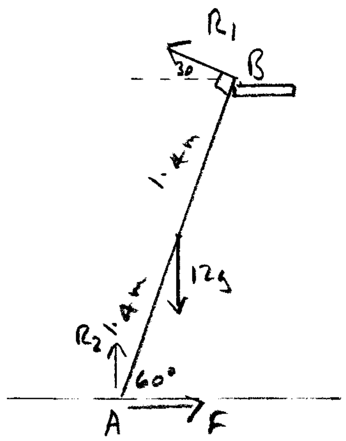
$T_{TB} \times 8 = 30g \times 4\sin 30° + 20g \times 8\sin 30°$

$8T_{TB} = 120g \sin 30° + 160g \sin 30°$

$8T_{TB} = 60g + 80g = 140g$

$T_{TB} = 17.5g = 171.5 \text{ N}$

13)



i)

Moments about A

$12g \times 1.4\cos 60 = 2.8R_1$

$8.4g = 2.8R_1$

$\Rightarrow R_1 = 3g \text{ N}$

ii)

Resolving horizontally

$F = R_1 \cos 30°$

$= 25.5 \text{ N}$

$R_2 + R_1 \sin 30° = 12g$

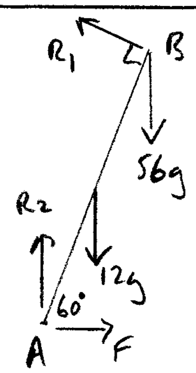
$R_2 = 12g - R_1 \sin 30$

$R_2 = 12g - 3g \times \frac{1}{2}$

$R_2 = \frac{21g}{2} = 102.9 \text{ N}$

$R_2 = 103 \text{ N}$ to 3 s.f.

13iii)



Suppose Jules at top of ladder

Moments about A

$2.8R_1 = 12g \times 1.4\cos 60 + 56g \times 2.8\cos 60$

$\Rightarrow R_1 = 303.8 \text{ N}$

Now $R_2 = 12g + 56g - R_1 \sin 30$

$\Rightarrow R_2 = 514.5 \text{ N}$

To stop slipping

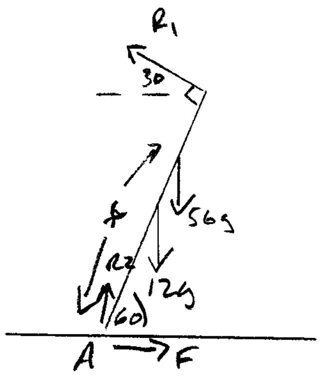
$F_{max} = \mu R_2 \geqslant R_1 \cos 30$

$\mu \times 514.5 \geqslant 303.8\cos 30$

$\mu \geqslant \frac{303.8\cos 30}{514.5}$

$\mu \geqslant 0.511$

13iv)



$\mu = 0.4$

Moments about A

$$12g \times 1.4\cos 60 + 56g \times x\cos 60 = R_1 \times 2.8$$

$$8.4g + 28gx = 2.8R_1 \quad (*)$$

Now $R_2 = 12g + 56g - R_1 \sin 30$

$$R_2 = 68g - 0.5R_1$$

$\therefore F_{max} = \mu R_2$

$$= 0.4(68g - 0.5R_1)$$

$$\Rightarrow R_1 \cos 30 = 0.4(68g - 0.5R_1)$$

$$R_1 \cos 30 = 27.2g - 0.2R_1$$

$$R_1(\cos 30 + 0.2) = 27.2g$$

$$R_1 = \frac{27.2 \times 9.8}{\cos 30 + 0.2}$$

$$R_1 = 250.1 \text{ N}$$

Subst for R_1 in (*)

$$8.4g + 28gx = 2.8 \times 250.1$$

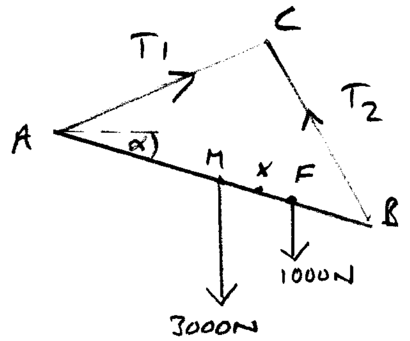
$$28gx = 2.8 \times 250.1 - 8.4g$$

$$x = \frac{2.8 \times 250.1 - 8.4g}{28g}$$

$$x = 2.25 \text{ m}$$

Jules can stand 2.25 m up ladder

14)
i)



AM = MB = 3m AF = 4m, BF = 2m

14 ii) Moments about C

$$3000 \times MX\cos\alpha = 1000 \times FX\cos\alpha$$

$$\Rightarrow 3MX = FX$$

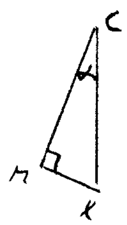
Total distance $MF = MB - BF$
$= 3 - 2 = 1\text{m}$

So $MX = 0.25$ m
$FX = 0.75$ m

$CM^2 = AC^2 - AM^2$ (Pythagoras)

$$CM^2 = 5^2 - 3^2 = 16$$

$$CM = 4\text{m}$$

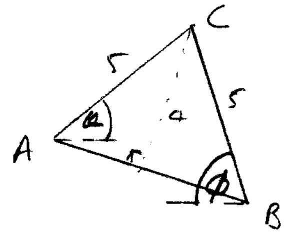


$$\tan\alpha = \frac{MX}{CM} = \frac{0.25}{4}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{0.25}{4}\right)$$

$$\alpha = 3.6°$$

14 iii)



$\angle CAB = \angle CBA = \sin^{-1}\left(\frac{4}{5}\right) = 53.1^\circ$

$\theta = 53.1 - \alpha = 53.1 - 3.6 = 49.5^\circ$

$\phi = 53.1 + \alpha = 53.1 + 3.6 = 56.7^\circ$

Resolving vertically

$T_1 \sin\theta + T_2 \sin\phi = 4000\ N$

Resolving horizontally

$T_1 \cos\theta = T_2 \cos\phi$

$\therefore\ T_1 = \dfrac{T_2 \cos\phi}{\cos\theta}$

Subst for T_1

$T_2 \tan\theta \cos\phi + T_2 \sin\phi = 4000$

$T_2 = \dfrac{4000}{\tan\theta\cos\phi + \sin\phi}$

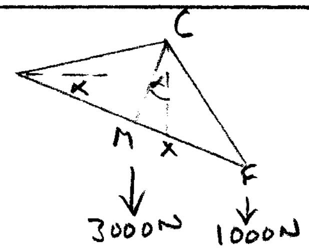
$T_2 = \dfrac{4000}{\tan 49.5^\circ \cos 56.7^\circ + \sin 56.7^\circ}$

$T_2 = 2705\ N$

$\Rightarrow T_1 = \dfrac{2705 \times \cos 56.7^\circ}{\cos 49.5^\circ}$

$T_1 = 2287\ N$

14 iv)



Moments about C

$3000\ MX \cos\alpha = 1000\ FX \cos\alpha$

$\Rightarrow\ 3MX = FX$

But $MX + FX = 3\ m$

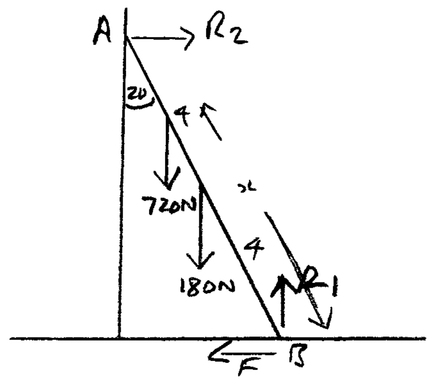
$\therefore\ MX = 0.75\ m$
$FX = 2.25\ m$

Now $\tan\alpha = \dfrac{MX}{CM} = \dfrac{0.75}{4}$

$\alpha = \tan^{-1}\left(\dfrac{0.75}{4}\right) = 10.6^\circ$

15)

i)



Vertically $R_1 = 720 + 180 = 900\ N$

Moments about A

$R_1 \times 8\sin 20^\circ = 720 \times (8 - x)\sin 20^\circ$
$+ 180 \times 4 \sin 20^\circ$
$+ F \times 8 \cos 20^\circ$

$\therefore\ 7200 \sin 20 = 5760 \sin 20$
$- 720x \sin 20$
$+ 720 \sin 20$
$+ 8F \cos 20^\circ$

15i) cont) $\therefore 8F\cos 20$

$= 7200\sin 20 - 5760\sin 20 + 720x\sin 20 - 720\sin 20$

$= \sin 20(720 + 720x)$

$\Rightarrow F = \dfrac{720\sin 20(1+x)}{8\cos 20}$

$F = 90\tan 20°(1+x)$

15ii)

As x increases
$(1+x)$ increases, other terms constant
$\Rightarrow F$ increases

To stand anywhere on ladder x can be up to 8 m

$\therefore F = 90\tan 20°(1+8)$

$F = 810\tan 20°$ N

$\therefore \mu R_1 \geq 810\tan 20°$

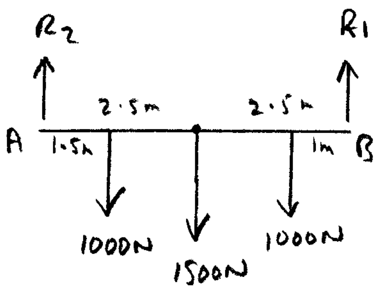
$\Rightarrow 900\mu \geq 810\tan 20°$

$\mu \geq \dfrac{810\tan 20°}{900}$

$\mu \geq 0.328$

16)

i)



Moments about A

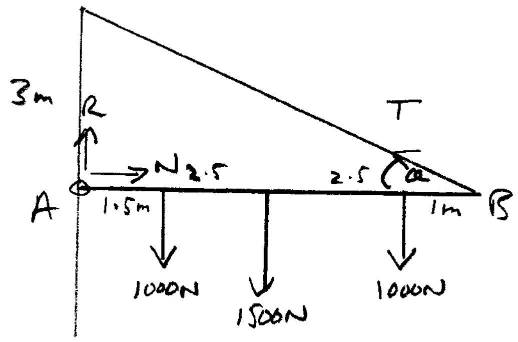
$5R_1 = 1000 \times 1.5 + 1500 \times 2.5 + 1000 \times 4$

$5R_1 = 1500 + 3750 + 4000$

$5R_1 = 9250$ Nm

$R_1 = 1850$ N

16ii)



$\tan\alpha = \dfrac{3}{5} \Rightarrow \alpha = 30.96°$

Moments about A

$T\sin\alpha \times 5 = 9250$ (from above)

$T = \dfrac{9250}{5\sin 30.96°}$

$T = 3596$ N

16iii)

Resolve vertically

$R + T\sin 30.96° = 3500$ N

$R = 3500 - 3596\sin 30.96°$

$\underline{R = 1650 \text{ N}}$

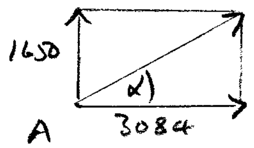
Resolve horizontally

$N = T\cos 30.96°$
$= 3596\cos 30.96° = 3084$ N

16iii) cont) Magnitude of force at A

$$= \sqrt{1650^2 + 3084^2}$$

$$= 3498 \text{ N}$$

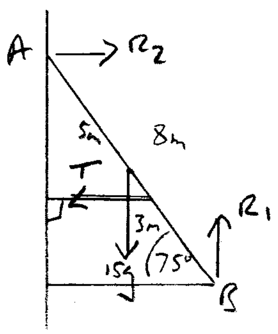


$$\alpha = \tan^{-1}\left(\frac{1650}{3084}\right)$$

$$\alpha = 28.1^\circ$$

Direction 28.1° above horizontal

17) i)



Resolve vertically $R_1 = 15g$

Moments about A

$$R_1 \times 8\cos 75 = 15g \times 4\cos 75 + T \times 5\sin 75$$

$$15g \times 8\cos 75 - 15g \times 4\cos 75 = T \times 5\sin 75$$

$$\Rightarrow T = \frac{60g\cos 75}{5\sin 75} = 31.51\text{N} \approx 31.5\text{ N}$$

17ii) Let man be x m up ladder

Moments about A

$$R_1 \times 8\cos 75 = 15g \times 4\cos 75 + 80g(8-x)\cos 75 + T \times 5\sin 75$$

This time $R_1 = 15g + 80g = 95g$

$\therefore$ Letting $T = 300\text{N}$ gives

$$95g \times 8\cos 75^\circ = 60g\cos 75 + 640g\cos 75 - 80gx\cos 75 + 300 \times 5\sin 75$$

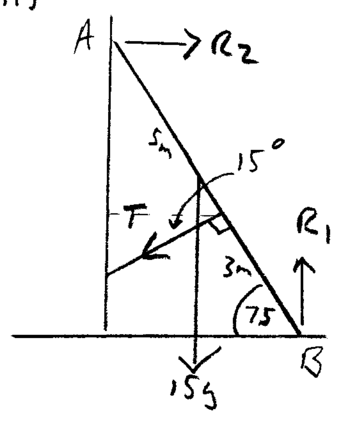
$$760g\cos 75 - 60g\cos 75 - 640g\cos 75 - 1500\sin 75 = -80gx\cos 75$$

$$\Rightarrow x = \frac{60g\cos 75 - 1500\sin 75}{-80g\cos 75}$$

$$x = 6.39 \text{ m}$$

Can climb 6.39 m up ladder before rope breaks

17iii)



17iii) cont) Resolve vertically

$$15g + T\sin 15^\circ = R_1$$

Moments about A

$$R_1 \times 8\cos 75 = 15g \times 4\cos 75 + 5T$$

$$(15g + T\sin 15^\circ) \times 8\cos 75^\circ$$

$$= 60g\cos 75^\circ + 5T$$

$$120g\cos 75 + 8T\sin 15^\circ\cos 75^\circ$$

$$= 60g\cos 75^\circ + 5T$$

$$60g\cos 75 = T(5 - 8\sin 15^\circ\cos 75^\circ)$$

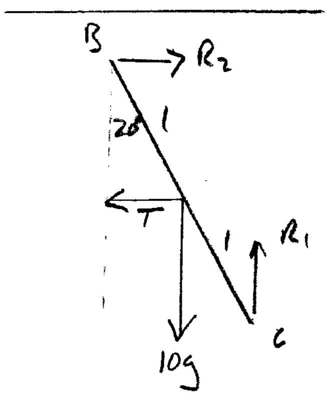
$$T = \frac{60g\cos 75^\circ}{5 - 8\sin 15^\circ\cos 75^\circ}$$

$$T = 34.1 \text{ N}$$

18)

i) Due to symmetry, the internal vertical forces on the rods at B are equal, but they must be opposite, and are $\therefore$ zero.

ii)



iii) Resolving vertically

$$R_1 = 10g$$

Moments about B

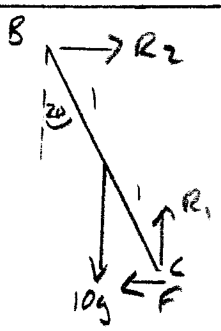
$$R_1 \times 2\sin 20^\circ = 10g \times 1\sin 20^\circ + T \times 1\cos 20^\circ$$

$$10g \times 2\sin 20^\circ - 10g \times 1\sin 20^\circ = T\cos 20^\circ$$

$$T = \frac{10g\sin 20}{\cos 20^\circ}$$

$$T = 35.7 \text{ N}$$

18iv)



Again $R_1 = 10g$

Moments about B

$$R_1 \times 2\sin 20^\circ = 10g \times 1\sin 20^\circ + F \times 2\cos 20^\circ$$

$$10g\sin 20^\circ = 2F\cos 20^\circ$$

$$F = \frac{10g\sin 20^\circ}{2\cos 20^\circ}$$

$$\mu R_1 \geq 5g\tan 20^\circ$$

$$\mu \geq \frac{5g\tan 20^\circ}{10g}$$

$$\mu \geq \frac{\tan 20^\circ}{2}$$

$$\mu \geq 0.182$$