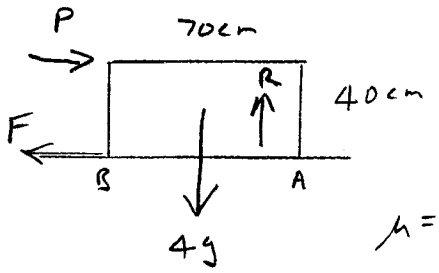
MEI MECHANICS 2 SLIDING AND TOPPLING EXERCISE 3C

1)



$\mu = 0.7$

i) On point of sliding

$P = F = \mu R = 0.7 \times 4g$

$P = 27.44$ N

$P = 27.4$ N

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ii) On point of toppling R acts through point A.

Moments about A

$P \times 0.4 = 4g \times 0.35$

$P = \frac{4g \times 0.35}{0.4} = 34.3$ N

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iii) Block slides since 27.4 N is reached before 34.3 N

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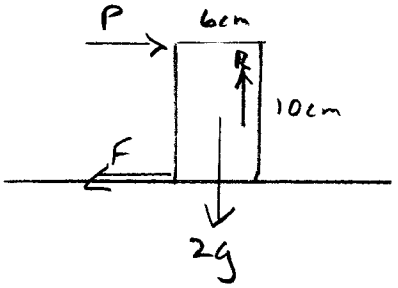
2)



i) On point of sliding

$P = F = \mu R = \mu \times 2g$

$\therefore P = 2g\mu$

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ii) If on point of toppling about A, then R acts through A.

Moments about A

$P \times 0.1 = 2g \times 0.03$

$\Rightarrow P = \frac{2g \times 0.03}{0.1}$

$P = 0.6g = 5.88$ N

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iii) For sliding $2g\mu < 0.6g$

$\Rightarrow \mu < \frac{0.6g}{2g}$

$\Rightarrow \mu < 0.3$

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iv) For toppling $0.6g < 2g\mu$

$\Rightarrow \mu > 0.3$

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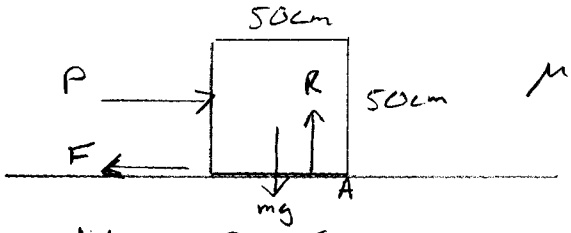
3)



$\mu = 0.4$

For sliding $P = F = \mu R$

$P = 0.4mg$

For toppling R acts through A

$P \times 0.25 = mg \times 0.25$

$\Rightarrow P = mg$

Cube will slide since $0.4mg < mg$

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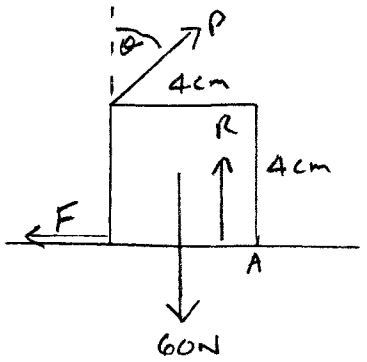
4)



$\mu = 0.4$

i) a) $\theta = 60°$

On point of sliding

$P\sin\theta = F = \mu R$

$= \mu(60 - P\cos\theta)$

$P\sin\theta = 0.4(60 - P\cos\theta)$

$P\sin\theta = 24 - 0.4P\cos\theta$

$P(\sin\theta + 0.4\cos\theta) = 24$

$$P = \frac{24}{\sin\theta + 0.4\cos\theta}$$

$\theta = 60°$

$$P = \frac{24}{\sin 60 + 0.4\cos 60}$$

$P = 22.5$ N

b) $\theta = 80°$

$$P = \frac{24}{\sin 80 + 0.4\cos 80}$$

$P = 22.8$ N

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ii) On point of toppling R acts through A

Moments about A

Resolve P vertically and horizontally

$P\cos\theta \times 0.04 + P\sin\theta \times 0.04 = 60 \times 0.02$

$$P = \frac{60 \times 0.02}{0.04(\sin\theta + \cos\theta)}$$

a) When $\theta = 60°$

$$P = \frac{60 \times 0.02}{0.04(\sin 60 + \cos 60)}$$

$P = 22.0$ N

b) When $\theta = 80°$

$$P = \frac{60 \times 0.02}{0.04(\sin 80 + \cos 80)}$$

$P = 25.9$ N

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iii) a) When $\theta = 60°$

Cube topples since $22.0 < 22.5$

b) When $\theta = 80°$

Cube slides since $22.8 < 25.9$

Both topple and slide when

$$P = \frac{24}{\sin\theta + 0.4\cos\theta} = \frac{60 \times 0.02}{0.04(\sin\theta + \cos\theta)}$$

$$\frac{24}{\sin\theta + 0.4\cos\theta} = \frac{30}{\sin\theta + \cos\theta}$$

4iii cont)

$24\sin\theta + 24\cos\theta = 30\sin\theta + 12\cos\theta$

$24\cos\theta - 12\cos\theta = 30\sin\theta - 24\sin\theta$

$12\cos\theta = 6\sin\theta$

$\frac{12}{6} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

$\Rightarrow \tan\theta = 2$

$\Rightarrow \theta = \tan^{-1}2 = 63.4^\circ$

For this value

$$P = \frac{30}{\sin 63.43^\circ + \cos 63.43^\circ}$$

$P = 22.4\ N$

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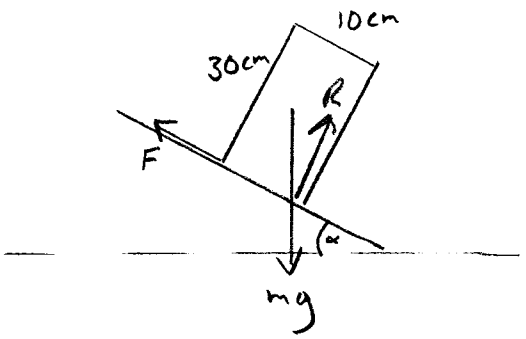
5)



At point of sliding

$mg\sin\alpha = F = \mu R$

$mg\sin\alpha = \mu mg\cos\alpha$

$\frac{mg\sin\alpha}{mg\cos\alpha} = \mu$

$\mu = \tan\alpha$

$\alpha = \tan^{-1}(0.25)$

$\alpha = 14.0^\circ$

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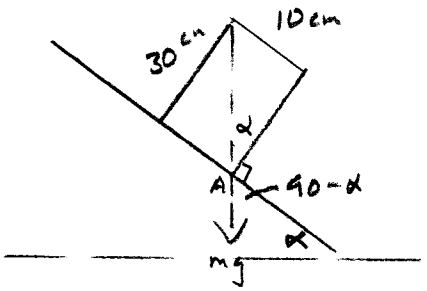
ii) At point of toppling



Weight acts through pivot point A

$\tan\alpha = \frac{10}{30}$

$\alpha = \tan^{-1}\left(\frac{1}{3}\right) = 18.4^\circ$

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iii) Slides first since $14.0^\circ < 18.4^\circ$

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6)

i) a) least likely to topple with 50cm x 20cm on slope

b) most likely to topple with 10cm x 20cm on slope

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ii) Not at all if faces identical

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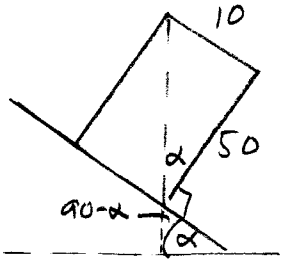
iii)



Would topple if

$\alpha > \tan^{-1}\left(\frac{10}{50}\right) = 11.3^\circ$

To ensure sliding, must have

$\mu < \tan 11.3^\circ = \frac{1}{5}$

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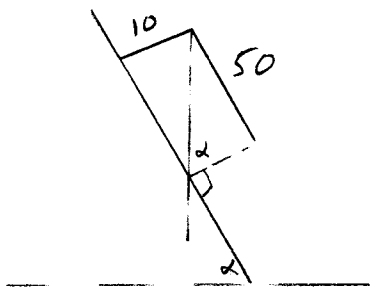

For toppling $\alpha = \tan^{-1} 5$

$\alpha = 78.7°$

$\therefore$ we require $\mu > 5$

for toppling

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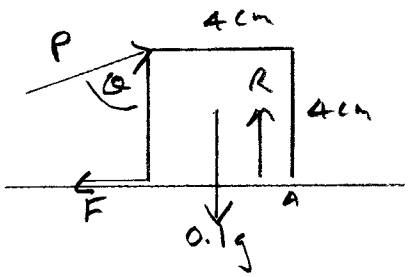
7)



$\mu = 0.3$

At point of sliding

$P\sin\theta = F = \mu R$

$P\sin\theta = \mu(0.1g - P\cos\theta)$

$P\sin\theta + 0.3P\cos\theta = 0.03 \times 9.8$

$P = \dfrac{0.03 \times 9.8}{\sin\theta + 0.3\cos\theta}$

when $\theta = 45°$

$P = \dfrac{0.03 \times 9.8}{\sin 45° + 0.3\cos 45°}$

$P = 0.3198\ N$

at point of sliding

Since we are given $P = 0.3N$, cube does not slide

At point of toppling R acts through A

Moments about A

$P\sin\theta \times 0.04 + P\cos\theta \times 0.04 = 0.1g \times 0.02$

$P = \dfrac{0.1g \times 0.02}{0.04(\sin\theta + \cos\theta)}$

$P = \dfrac{0.05 \times 9.8}{\sin\theta + \cos\theta}$

when $\theta = 45°$

$P = 0.3465\ N$

Since we are given $P = 0.3N$ cube does not topple

$\therefore$ cube remains stationary

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ii) If $\theta = 15°$

For sliding $P = \dfrac{0.3 \times 9.8}{\sin 15° + 0.3\cos 15°}$

$P = 5.359\ N$

Given $P = 0.45$ $\therefore$ no sliding

For toppling $P = \dfrac{0.05 \times 9.8}{\sin 15° + \cos 15°}$

$P = 0.400\ N$

Given that $P = 0.45N$

$\therefore$ cube topples

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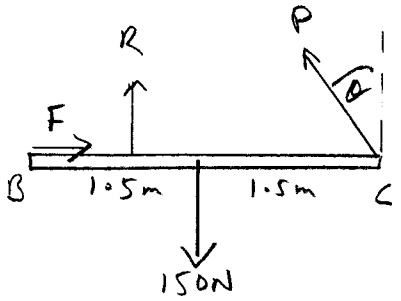
8)



On point of sliding

i) Resolve vertically

$$R + P\cos\theta = 150 \text{ N}$$

$$R = (150 - P\cos\theta) \text{ N}$$

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ii) On point of sliding

$$P\sin\theta = F = \mu R$$

$$P\sin\theta = \mu(150 - P\cos\theta)$$

$$P\sin\theta = 150\mu - \mu P\cos\theta$$

$$P\sin\theta + \mu P\cos\theta = 150\mu$$

$$P(\sin\theta + \mu\cos\theta) = 150\mu$$

$$P = \frac{150\mu}{(\sin\theta + \mu\cos\theta)}$$

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iii) On point of turning:
Normal reaction acts vertically through a point on edge AB

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iv) Moments about B

$$P\cos\theta \times 3 = 150 \times 1.5$$

$$P = \frac{150 \times 1.5}{3\cos\theta} = \frac{75}{\cos\theta}$$

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v) Slides before toppling

$$\Rightarrow \frac{150\mu}{\sin\theta + \mu\cos\theta} < \frac{75}{\cos\theta}$$

$$150\mu\cos\theta < 75\sin\theta + 75\mu\cos\theta$$

$$150\mu\cos\theta - 75\mu\cos\theta < 75\sin\theta$$

$$75\mu\cos\theta < 75\sin\theta$$

$$\mu < \frac{75\sin\theta}{75\cos\theta}$$

$$\Rightarrow \mu < \tan\theta$$

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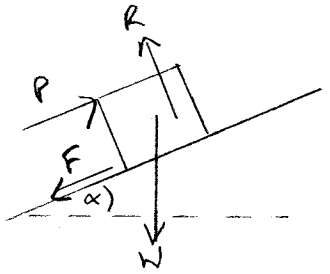
9)



i) Resolving ⊥ to slope

$$R = W\cos\alpha$$

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ii) Will slip when $P > W\sin\alpha + F_{max}$

$$P > W\sin\alpha + \mu R$$

$$P > W\sin\alpha + \mu W\cos\alpha$$

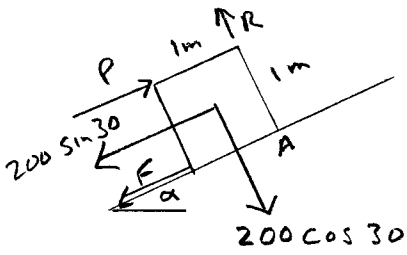
If $W = 200$ N and $\alpha = 30°$

Slip if $P > 200(\sin 30 + \mu\cos 30)$

$$P > 200\left(\frac{1}{2} + \mu\frac{\sqrt{3}}{2}\right)$$

$$P > 100(1 + \sqrt{3}\mu)$$

9iii)



On point of turning R acts through a point on AB

Resolve weight parallel and ⊥ to slope

Then moment of weight about line AB

$= 200\sin 30 \times 0.5 + 200\cos 30 \times 0.5$

$= 100(\sin 30 + \cos 30)$

$= 136.6$ Nm

Will turn when

$P \times 1 > 136.6$ Nm

$\Rightarrow \quad P > 136.6$ N

9iv)

For no topple $P < 136.6$ N

To slip $P > 100(1 + \sqrt{3}\mu)$

$\therefore \quad 100(1 + \sqrt{3}\mu) < 136.6$

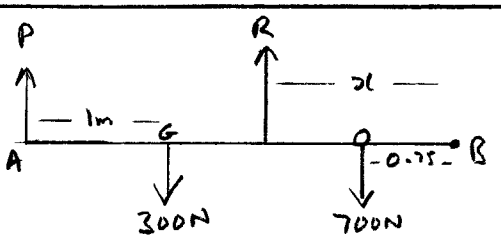
$1 + \sqrt{3}\mu < 1.366$

$\sqrt{3}\mu < 0.366$

$\mu < 0.211$

10)

i)



Moments about B

$P \times 3 + R x = 300 \times 2 + 700 \times 0.75$

$3P + Rx = 1125$ Nm (1)

Resolving vertically

$P + R = 300 + 700$

$\Rightarrow \quad P = 1000 - R$

From (1) $R = \dfrac{1125 - 3P}{x}$

so $P = 1000 - \dfrac{(1125 - 3P)}{x}$

$\Rightarrow \quad Px = 1000x - 1125 + 3P$

$\Rightarrow \quad 1125 - 1000x = 3P - Px$

$\Rightarrow \quad 1125 - 1000x = P(3 - x)$

$\Rightarrow \quad P = \dfrac{1125 - 1000x}{3 - x}$

$= \quad P = \dfrac{125(9 - 8x)}{3 - x}$

10 ii)

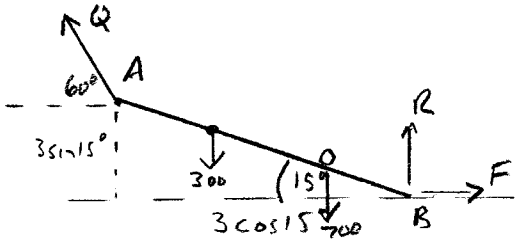
At point of turning R acts through B $\therefore \quad x = 0$

Moments about B

$3P + 0 = 1125$

$P = 375$ N

10iii)



Taking moments about B with Q resolved vertically and horizontally

$$300 \times 2\cos 15^\circ + 700 \times 0.75\cos 15^\circ + Q\cos 60 \times 3\sin 15^\circ = Q\sin 60 \times 3\cos 15^\circ$$

$$1086.67 = Q(\sin 60 \times 3\cos 15 - \cos 60 \times 3\sin 15^\circ)$$

$$\Rightarrow Q = \frac{1086.67}{(\sin 60 \times 3\cos 15^\circ - \cos 60 \times 3\sin 15^\circ)}$$

$$Q = 512.26 \text{ N}$$

$$Q \approx 512 \text{ N}$$

On point of slipping so

$$F = \mu R = Q\cos 60$$

$$\Rightarrow \mu = \frac{512.26 \times \cos 60}{R}$$

But $R = 1000 - Q\sin 60$

$$\therefore \mu = \frac{512.26 \times \cos 60}{1000 - 512.26\sin 60}$$

$$\mu = 0.460$$