MEI MECHANICS 2 COEFFICIENT OF RESTITUTION EXERCISE 6C

1) i) $e = \frac{V_S}{V_A} = \frac{1.2}{1.8} = \frac{2}{3}$

ii) $\frac{V_S}{V_A} = e \qquad \frac{V_S}{2.4} = 0.6$

$V_S = 0.6 \times 2.4$

$= 1.44\ ms^{-1}$

iii) $e = \frac{V_S}{V_A} = \frac{1.8}{2.4} = 0.75$

iv) $\frac{V_S}{V_A} = e \qquad \frac{V_S}{4} = 0.8$

$V_S = 0.8 \times 4 = 3.2\ ms^{-1}$

iii) loss in KE

$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 0.06(15^2 - 12^2)$

$= 2.43\ J$

2) i) $e = \frac{V_S}{V_A} = \frac{3}{10} = 0.3$

ii) $e = \frac{V_S}{V_A} = \frac{0}{5} = 0$

iii) $e = \frac{V_S}{V_A} = \frac{7.6}{8} = 0.95$

iv) $e = \frac{V_S}{V_A} = \frac{3 \times 10^8}{3 \times 10^8} = 1$

3) i) $e = \frac{V_S}{V_A} = \frac{12}{15} = 0.8$

ii) Change in momentum = Impulse

$J = mv - mu$

$J = 0.06(12 - -15)$

$J = 0.06(27) = 1.62\ Ns$

4) i) $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 9.8 \times 1$

$v = 4.43\ ms^{-1}$

ii) Bounces up to 0.81 m

Using $v^2 = u^2 + 2as$

$0 = u^2 - 2 \times 9.8 \times 0.81$

$u^2 = 2 \times 9.8 \times 0.81$

$u = 3.98\ ms^{-1}$

iii)

$e = \frac{3.98447}{4.42719}$

$e = 0.9$

iv) Change in K.E

$= \frac{1}{2}m(v^2 - u^2)$

$= \frac{1}{2} \times 0.08 \times (4.427^2 - 3.984^2)$

$= 0.149\ J$

4v) Change in gpe

$= mg(1-0.81)$

$= 0.08 \times 9.8 \times 0.19$ J

$= 0.149$ J

4vi)

height of next bounce

$= 0.81 \times e^2$

$= 0.81 \times 0.9^2$

$= 0.656$ m

5)

i) $m_1U_1 + m_2U_2 = m_1V_1 + m_2V_2$

$150(3) + 150(-2) = 150(0) + 150V_2$

$450 - 300 = 0 + 150V_2$

$150 = 150V_2$

$V_2 = 1 \text{ ms}^{-1}$

Stuart travelling 1 ms^{-1} in Isobels original direction

ii)

$e = \dfrac{V_S}{V_A} = \dfrac{1}{5} = 0.2$

iii)

Impulse J = Change in momentum

$J = mV - mU$

$= 150(1 - -2)$

$= 450$ Ns

iv) Original ke

$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$= 75 \times 3^2 + 75 \times 2^2 = 975$ J

Final ke

$= 0 + \frac{1}{2} \times 150 \times 1^2 = 75$ J

Loss in k.e. = 900 J

6)

i) $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 10 \times 20$

$v = 20 \text{ ms}^{-1}$

ii)

$e = \dfrac{V_S}{V_A} = \dfrac{15}{20} = 0.75$

iii)

Impulse = change in momentum

$= mv - mu$

$= 50(15 - -20)$

$= 1750$ Ns

iv)

lost ke $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2}m(u^2 - v^2)$

$= \frac{1}{2} \times 50(20^2 - 15^2)$

$= 4375$ J

v) low - do not want to bounce high with a large impulse

7) a) $\overset{4}{\rightarrow}$ $\overset{2}{\rightarrow}$ ⑤ ⑤ $e = \frac{1}{2}$ $\overset{V_1}{\rightarrow}$ $\overset{V_2}{\rightarrow}$ ⑤ ⑤

$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$$

$$20 + 10 = 5V_1 + 5V_2$$

$$6 = V_1 + V_2 \quad (1)$$

Also $\dfrac{V_2 - V_1}{2} = \dfrac{1}{2}$

$$V_2 - V_1 = 1$$

$$V_2 = V_1 + 1 \quad (2)$$

Subst in (1)

$$6 = V_1 + V_1 + 1$$

$$2V_1 = 5$$

$$\underline{V_1 = 2.5 \text{ ms}^{-1}}$$

$$\underline{V_2 = 3.5 \text{ ms}^{-1}}$$

Loss in K.E.

$$\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 - \left(\tfrac{1}{2} m_1 V_1^2 + \tfrac{1}{2} m_2 V_2^2\right)$$

$$= 2.5 \times 4^2 + 2.5 \times 2^2 - \left(2.5 \times 2.5^2 + 2.5 \times 3.5^2\right)$$

$$= 3.75 \text{ J}$$

7b) $\overset{4}{\rightarrow}$ $\overset{2}{\leftarrow}$ ⑤ ⑤ $e = \frac{1}{2}$ $V_1 \rightarrow$ $\rightarrow V_2$ ⑤ ⑤

$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$$

$$20 - 10 = 5V_1 + 5V_2$$

$$2 = V_1 + V_2 \quad (1)$$

Also $\dfrac{V_2 - V_1}{6} = \dfrac{1}{2}$

$$V_2 - V_1 = 3 \quad (2)$$

(1)+(2)

$$2V_2 = 5$$

$$\underline{V_2 = 2.5 \text{ ms}^{-1}}$$

Subst in (2) $2.5 - V_1 = 3$

$$\underline{V_1 = -0.5 \text{ ms}^{-1}}$$

or $\overset{0.5}{\leftarrow}$ $\overset{2.5}{\rightarrow}$ ⑤ ⑤

loss in k.e.

$$\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 - \left(\tfrac{1}{2} m_1 V_1^2 + \tfrac{1}{2} m_2 V_2^2\right)$$

$$= 2.5 \times 4^2 + 2.5 \times 2^2 - \left(2.5 \times 2.5^2 + 2.5 \times (-0.5)^2\right)$$

$$= 33.75 \text{ J}$$

7c) $\overset{2}{\rightarrow}$ $\overset{0}{\rightarrow}$ ⑥ ④ $e = 0$ $\overset{V}{\rightarrow}$ ○○

$e = 0 \Rightarrow$ no separation

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$$

$$12 + 0 = 10V$$

$$\underline{V = 1.2 \text{ ms}^{-1}}$$

Loss in KE $= \tfrac{1}{2} m_1 u_1^2 - \tfrac{1}{2}(m_1 + m_2) V^2$

$$= \tfrac{1}{2} \times 6 \times 2^2 - \tfrac{1}{2} \times 10 \times 1.2^2$$

$$= 4.8 \text{ J}$$

7d)

$\overset{1}{\rightarrow}$ $\overset{2}{\leftarrow}$ $\quad$ $\overset{V_1}{\rightarrow}$ $\overset{V_2}{\rightarrow}$

(2) (1) $e=1$ (2) (1)

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$2 - 2 = 2V_1 + V_2$$

$$0 = 2V_1 + V_2 \qquad (1)$$

Also $\dfrac{V_2 - V_1}{3} = 1$

$$V_2 - V_1 = 3 \qquad (2)$$

(1) - (2) $\quad 3V_1 = -3$

$$\underline{V_1 = -1 \text{ ms}^{-1}}$$

Subst in (2)

$$V_2 - -1 = 3$$

$$\underline{V_2 = 2 \text{ ms}^{-1}}$$

loss in K.E $= 0$
since $e = 1$, elastic collision

7e)

$\overset{1}{\rightarrow}$ $\overset{2}{\leftarrow}$ $\quad$ $\rightarrow^{V_1}$ $\rightarrow^{V_2}$

(2) (1) $e=\frac{1}{2}$ (2) (1)

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$2 - 2 = 2V_1 + V_2$$

$$0 = 2V_1 + V_2 \qquad (1)$$

Also

$$\frac{V_2 - V_1}{3} = \frac{1}{2}$$

$$V_2 - V_1 = \frac{3}{2} \qquad (2)$$

(1) - (2) $\quad 3V_1 = -\frac{3}{2}$

$$\underline{V_1 = -\frac{1}{2} \text{ ms}^{-1}}$$

Subst in (2)

$$V_2 - -\frac{1}{2} = \frac{3}{2}$$

$$V_2 + \frac{1}{2} = \frac{3}{2}$$

$$\underline{V_2 = 1 \text{ ms}^{-1}}$$

Loss in k.e

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 - \left(\tfrac{1}{2}m_1V_1^2 + \tfrac{1}{2}m_2V_2^2\right)$$

$$= 1(1)^2 + \tfrac{1}{2}(2)^2 - \left(1\left(-\tfrac{1}{2}\right)^2 + \tfrac{1}{2}(1)^2\right)$$

$$= 2.25 \text{ J}$$

7f)

$\overset{8}{\rightarrow}$ $\overset{2}{\leftarrow}$ $\quad$ $\overset{V_1}{\rightarrow}$ $\overset{V_2}{\rightarrow}$

(4) (4) $e=0.2$ (4) (4)

$$m_1u_1 + m_2u_2 = m_1V_1 + m_2V_2$$

$$32 - 8 = 4V_1 + 4V_2$$

$$6 = V_1 + V_2 \qquad (1)$$

Also $\dfrac{V_2 - V_1}{10} = 0.2$

$$V_2 - V_1 = 2 \qquad (2)$$

(1)+(2)

$$2V_2 = 8$$

$$\underline{V_2 = 4 \text{ ms}^{-1}}$$

Subst in (1) $\quad \underline{V_1 = 2 \text{ ms}^{-1}}$

Loss in ke $= \frac{1}{2} \times 4\left(8^2 + 2^2 - 4^2 - 2^2\right)$

$$= 96 \text{ J}$$

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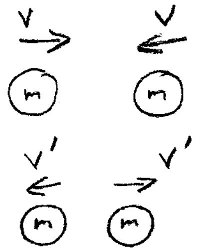
8) i)



ii) $e = \dfrac{2V'}{2V} \Rightarrow V' = eV$

iii) K.E. lost =

$\frac{1}{2}mV^2 + \frac{1}{2}mV^2 - \left(\frac{1}{2}mV'^2 + \frac{1}{2}mV'^2\right)$

$= mV^2 - mV'^2$

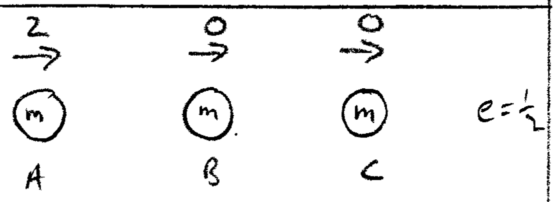
$= mV^2 - m(eV)^2$

$= mV^2 - me^2V^2$

$= mV^2(1-e^2)$

iv) $e \leq 1$ otherwise a collision could result in a gain of kinetic energy

9)



Collision A and B

$m_A U_1 + m_B U_2 = m_A V_1 + m_B V_2$

$2m + 0 = mV_1 + mV_2$

$2 = V_1 + V_2$ (1)

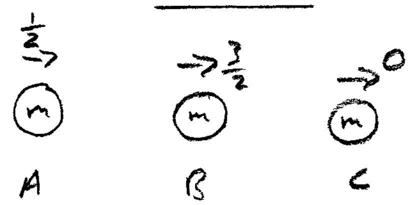
Also $\dfrac{V_2 - V_1}{2} = \dfrac{1}{2}$

$V_2 - V_1 = 1$ (2)

(1)+(2) $2V_2 = 3$

$V_2 = \frac{3}{2}$

$V_1 = \frac{1}{2}$



Collision B and C

$\frac{3}{2}m + 0 = mV_2 + mV_3$

$\frac{3}{2} = V_2 + V_3$ (3)

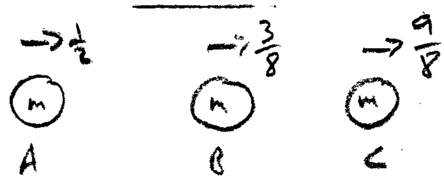
Also $\dfrac{V_3 - V_2}{\frac{3}{2}} = \dfrac{1}{2}$

$V_3 - V_2 = \frac{3}{4}$ (4)

(3)+(4) $2V_3 = \frac{9}{4}$

$V_3 = \frac{9}{8}$

$V_2 = \frac{3}{8}$



Collision A and B again

$\frac{1}{2}m + \frac{3}{8}m = mV_1 + mV_2$

$\frac{7}{8} = V_1 + V_2$ (5)

Also $\dfrac{V_2 - V_1}{\frac{1}{8}} = \dfrac{1}{2}$

$V_2 - V_1 = \frac{1}{16}$ (6)

9 cont) 5 + 6 $2v_2 = \frac{15}{16}$

$v_2 = \frac{15}{32}$

$v_1 = \frac{13}{32}$

$\xrightarrow{13/32}$ (m) A $\xrightarrow{15/32}$ (m) B $\xrightarrow{9/8}$ (m) C

10) $\xrightarrow{8}$ (m) A $\xrightarrow{0}$ (m) B $e = 0.75$

First collision

$8m + 0 = mV_A + mV_B$

$8 = V_A + V_B$ (1)

Also $\frac{V_B - V_A}{8} = 0.75$

$V_B - V_A = 6$ (2)

(1) + (2)

$2V_B = 14$

$\underline{V_B = 7 \text{ ms}^{-1}}$

$\underline{V_A = 1 \text{ ms}^{-1}}$

$\xrightarrow{1}$ (m) A $\xrightarrow{7}$ (m) | $e = \frac{1}{2}$

B Hits cushion rebounds at $7 \times \frac{1}{2}$ ms^{-1}

$\xrightarrow{1}$ (m) A $\xleftarrow{3.5}$ (m) B

2nd collision between A and B

$1m - 3.5m = mV_A + mV_B$

$-2.5 = V_A + V_B$ (3)

Also $\frac{V_B - V_A}{4.5} = 0.75$

$V_B - V_A = 3.375$ (4)

(3) + (4)

$2V_B = 0.875$

$\underline{V_B = 0.4375 \text{ ms}^{-1}}$

$\underline{V_A = -2.9375 \text{ ms}^{-1}}$

11) i)

Before $\xrightarrow{3 \text{ ms}^{-1}}$ (40g) A $\xrightarrow{0 \text{ ms}^{-1}}$ (60g) B

After $\xrightarrow{0 \text{ ms}^{-1}}$ (40g) A $\xrightarrow{V_B}$ (60g) B

ii)

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$0.04 \times 3 + 0 = 0 + 0.06 V_B$

$\Rightarrow V_B = \frac{0.04 \times 3}{0.06} = 2 \text{ ms}^{-1}$

$V_B = 2 \text{ ms}^{-1}$

iii)

$e = \frac{V_S}{V_A} = \frac{2}{3}$

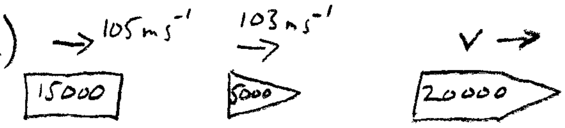
11 iv)
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 + 0 = 0 + m_2v_2$$
$$m_1u_1 = m_2v_2$$
$$\Rightarrow \frac{m_1}{m_2} = \frac{v_2}{u_1} = \frac{v_S}{v_A} = e$$
$$\therefore e = \frac{m_1}{m_2}$$

11 v) For this situation to occur where the approaching sphere becomes stationary $m_1 \le m_2$

11 vi)
$$\text{k.e. before} = \frac{1}{2}m_1u_1^2$$
$$\text{k.e. after} = \frac{1}{2}m_2v_2^2$$
$$\text{Ratio } \frac{\text{after}}{\text{before}} = \frac{\frac{1}{2}m_2v_2^2}{\frac{1}{2}m_1u_1^2} = \frac{m_2}{m_1}\left(\frac{v_2}{u_1}\right)^2 = \frac{1}{e} \times e^2 = e$$
$$\text{Ratio } \frac{\text{after}}{\text{before}} = e$$

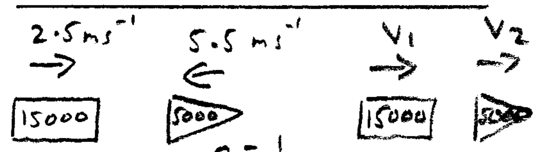
If k.e preserved then e must equal 1

12) i)



ii)
$$m_1u_1 + m_2u_2 = (m_1+m_2)v$$
$$15000 \times 105 + 5000 \times 103 = 20000v$$
$$v = \frac{2090000}{20000}$$
$$v = 104.5\ ms^{-1}$$

iii)



$e = \frac{1}{4}$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$15000 \times 2.5 - 5000 \times 5.5 = 15000v_1 + 5000v_2$$
$$10000 = 15000v_1 + 5000v_2$$
$$2 = 3v_1 + v_2 \quad (1)$$

Also
$$\frac{v_S}{v_A} = \frac{v_2 - v_1}{8} = \frac{1}{4} = e$$
$$v_2 - v_1 = \frac{8}{4} = 2 \quad (2)$$

(1) − (2) $\quad 4v_1 = 2 - 2 = 0$
$$\Rightarrow v_1 = 0\ ms^{-1}$$

∴ body comes to rest

Subst in (1) $\quad 2 = 0 + v_2$

$v_2 = 2\ ms^{-1}$ (forwards)

Nose travelling forwards at $2\ ms^{-1}$

12iv)

$$m_{fuel}\,u + m_{body}\,v = 0$$

$$m u + (15000 - m)v = 0$$

$$m \times -2000 + (15000-m)2.5 = 0$$

$$-2000m + 37500 - 2.5m = 0$$

$$37500 = 2002.5\,m$$

$$m = 18.7 \text{ kg}$$

13)

i) Using $v^2 = u^2 + 2as$

$$v^2 = 0 + 2gh$$

$$v = \sqrt{2gh}$$

Using $v = u + at$

$$\sqrt{2gh} = 0 + gt$$

$$t = \frac{\sqrt{2gh}}{g} = \sqrt{\frac{2h}{g}}$$

ii) Using $v^2 = u^2 + 2as$

$$0 = (e\sqrt{2gh})^2 - 2gs$$

$$2gs = e^2 2gh$$

$$s = e^2 h$$

iii)

$$s = (e^2)^n h$$

$$s = e^{2n} h$$

iv)

For time between 1st and 2nd bounce use $s = ut + \frac{1}{2}at^2$

$$0 = e\sqrt{2gh}\,t - \frac{1}{2}gt^2$$

$$0 = t\left(e\sqrt{2gh} - \frac{gt}{2}\right)$$

$$\frac{gt}{2} = e\sqrt{2gh}$$

$$t = \frac{2e\sqrt{2gh}}{g} = 2e\sqrt{\frac{2h}{g}}$$

Total time elapsed until 2nd bounce

$$\sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}}$$

Time elapsed until n^{th} bounce

$$\sqrt{\frac{2h}{g}} + 2e\sqrt{\frac{2h}{g}} + 2e^2\sqrt{\frac{2h}{g}} + \dots + 2e^{n-1}\sqrt{\frac{2h}{g}}$$

$$= \sqrt{\frac{2h}{g}}\left(1 + 2e + 2e^2 + \dots + 2e^{n-1}\right)$$

v)

$$= \sqrt{\frac{2h}{g}}\left(-1 + 2 + 2e + 2e^2 + \dots + 2e^{n-1}\right)$$

$$= \sqrt{\frac{2h}{g}}\left(-1 + 2\left(\frac{1-e^n}{1-e}\right)\right)$$

As $n \to \infty$ (infinite number of bounces)

Time elapsed $\to \sqrt{\frac{2h}{g}}\left(-1 + \frac{2}{1-e}\right)$

which is a finite time

vi)

Distance travelled

$$= h + 2e^2 h + 2e^4 h + \dots$$

13 vi) cont

$$= -h + 2h + 2e^2h + 2e^4h + \ldots$$

$$= -h + \frac{2h}{1-e^2}$$

$$= h\left[\frac{2}{1-e^2} - 1\right]$$

$$= h\left[\frac{2-(1-e^2)}{1-e^2}\right]$$

$$= h\left[\frac{e^2+1}{1-e^2}\right]$$

14) i)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$mu + 0 = mv_1 + mv_2$$

$$\Rightarrow \quad u = v_1 + v_2 \qquad (1)$$

Also $\dfrac{V_S}{V_A} = e = 1$

$$\Rightarrow \frac{v_2 - v_1}{u} = 1$$

$$v_2 - v_1 = u \qquad (2)$$

(1) + (2)

$$2v_2 = 2u$$

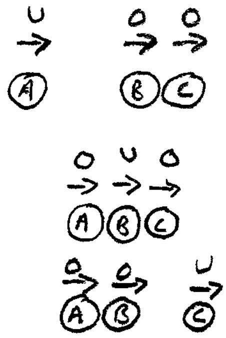
$$\Rightarrow \underline{v_2 = u}$$

Subst in (1)

$$u = v_1 + u$$

$$\Rightarrow v_1 = 0$$

ii)



A and B become stationary with C having velocity u

iii) A, B, C, D become stationary while E has velocity u

iv)

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15)

i) $V_A = 7\ \text{ms}^{-1}$

$V_S = eV_A = 0.6 \times 7 = 4.2\ \text{ms}^{-1}$

Impulse = Change in momentum

$= mv - mu$

$= 10(4.2 - -7)$

$= 112$ Ns

ii)

V_S after 3rd hit $= e^3 \times 7$

$= 0.6^3 \times 7 = 1.512\ \text{ms}^{-1}$

$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 10 \times 1.512^2$

$= 11.43072$ J

At top of rise, increase in gpe will equal lost ke

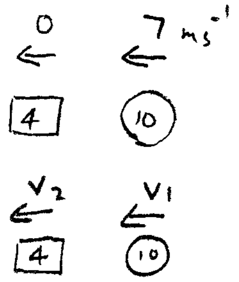
$mgh = 11.43072$

$h = \dfrac{11.43072}{10 \times 9.8}$

$h = 0.11664$ m

rise $h = 0.117$ m

iii)



$e = 0.4$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$10 \times 7 + 0 = 10v_1 + 4v_2$

$70 = 10v_1 + 4v_2$ (1)

Also $\dfrac{V_S}{V_A} = e$

$\dfrac{v_2 - v_1}{7} = 0.4$

$v_2 - v_1 = 2.8$ (2)

(2)×4 $4v_2 - 4v_1 = 11.2$ (3)

(1) − (3)

$58.8 = 14v_1$

$v_1 = 4.2\ \text{ms}^{-1}$

Sphere travels in original direction ie into wall at $4.2\ \text{ms}^{-1}$

16)

i) Impulse = Force × time

$= 15 \times 12$

$= 180$ Ns

Impulse = Change in momentum

$180 = mv - mu$

$180 = 20v - 20 \times 0$

$\dfrac{180}{20} = v$

$v = 9\ \text{ms}^{-1}$

16ii) The two particles form a closed system. Conservation of momentum applies. Since each particle is acted on by the other they do not form a closed system individually, therefore conservation of momentum does not apply to the individual particles

16iii)

$u_1 = 9, \quad u_2 = 0 \Rightarrow V_A = 9$

$v_1 = 3, \quad v_2 = 5 \Rightarrow V_S = 2$

$$e = \frac{V_S}{V_A} = \frac{2}{9}$$

16iv)

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$20 \times 9 + 0 = 20 \times 3 + 5m$

$180 - 60 = 5m$

$$m = \frac{120}{5} = 24 \text{ kg}$$

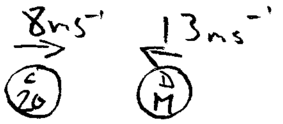
Impulse = Change in momentum

$= mv - mu$

$= 20(3 - 9) = -120 \text{ Ns}$

ie 120 Ns in opposite direction to original motion

16v)



$e = \frac{1}{3}$

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$20 \times 8 - 13M = 0 + MV_2$

$160 - 13M = MV_2$

$160 = M(V_2 + 13) \quad (*)$

Also $\frac{V_S}{V_A} = e \qquad \frac{V_2 - 0}{21} = \frac{1}{3}$

$$\Rightarrow V_2 = \frac{21}{3} = 7 \text{ ms}^{-1}$$

Subst in $(*)$

$160 = M(7 + 13)$

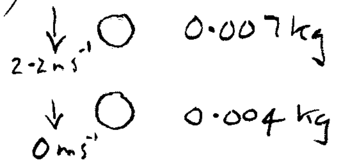
$\frac{160}{20} = M$

$$M = 8 \text{ kg}$$

If D > 8 kg, C would travel backwards to its original direction

17)

i)



$m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$

$0.007 \times 2.2 + 0 = 0.011V$

$$V = \frac{0.007 \times 2.2}{0.011}$$

$V = 1.4 \text{ ms}^{-1}$ downwards

17ii)

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$0.007 \times 2.2 + 0 = 0.007 \times 1 + 0.004 V_2$

$$V_2 = \frac{0.007 \times 2.2 - 0.007 \times 1}{0.004} = 2.1 \text{ ms}^{-1}$$

MEI MECHANICS 2 COEFFICIENT OF RESTITUTION EXERCISE 6C

17ii) Fly has velocity $2.1\,\text{ms}^{-1}$ downwards

$$e = \frac{V_S}{V_A} = \frac{2.1 - 1}{2.2} = \frac{1.1}{2.2} = \frac{1}{2}$$

$$e = \tfrac{1}{2}$$

17iii)

Conservation of linear momentum means impulses on each part are equal and opposite. ∴ movement is in vertical plane containing these impulse vectors.

17iv)

$$(m_1 + m_2)U = m_1 V_1 + m_2 V_2$$

$$0.007\begin{pmatrix}0\\1.5\end{pmatrix} = 0.005\begin{pmatrix}2.4\\0.7\end{pmatrix} + 0.002\begin{pmatrix}V_x\\V_y\end{pmatrix}$$

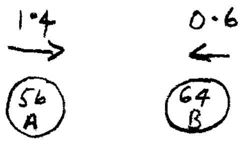
$$V_x = \frac{0 - 0.005 \times 2.4}{0.002} - 6$$

$$V_y = \frac{0.007 \times 1.5 - 0.005 \times 0.7}{0.002}$$

$$V_y = 3.5$$

Q has velocity $(-6\underline{i} + 3.5\underline{j})\,\text{ms}^{-1}$

18) i)



$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$$

$$56 \times 1.4 - 64 \times 0.6 = 56 \times 0.28 + 64 V_2$$

$$V_2 = \frac{56 \times 1.4 - 64 \times 0.6 - 56 \times 0.28}{64}$$

$V_2 = 0.38\,\text{ms}^{-1}$
in direction of Abi's original motion

18ii) $e = \dfrac{V_S}{V_A} = \dfrac{0.38 - 0.28}{2} = \dfrac{0.1}{2}$

$$e = 0.05$$

18iii)

Impulse = Change in momentum

$= m(v - u)$

$= 64(0.38 - -0.6)$

$= 62.72$ Ns

in direction of Abi's orriginal direction

18iv)

Impulse = Change in momentum

Abi's new momentum

$= 56 \times 0.28 + 4.48$ Ns

$= 20.16$ Ns

$mv = 20.16$

$56v = 20.16$

$$v = \frac{20.16}{56} = 0.36\,\text{ms}^{-1}$$

in original direction of Abi's motion

18v)

Momentum after embrace = momentum after 1st collision

$= 56 \times 0.28 + 64 \times 0.38$

$= 40$ Ns

Velocity of couple $(56 + 64)v = 40$

$$v = \frac{40}{120} = \tfrac{1}{3}\,\text{ms}^{-1}$$

18 v) cont)

K.E. after embrace

$= \frac{1}{2}(56+64)\times\left(\frac{1}{3}\right)^2$

$= 6.67 \text{ J}$

K.E. before 1st collision

$= \frac{1}{2}\times 56\times 1.4^2 + \frac{1}{2}\times 64\times 0.6^2$

$= 66.4 \text{ J}$

Lost K.E $= 66.4 - 6.67$

$= 59.73$

$= 59.7 \text{ J}$

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19) i) Using $v^2 = u^2 + 2as$

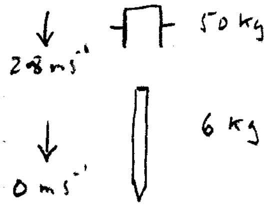
$v^2 = 0 + 2\times 9.8\times 0.4$

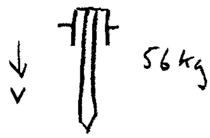
$v^2 = 7.84$

$v = 2.8 \text{ ms}^{-1}$

---

ii)





$m_1u_1 + m_2u_2 = (m_1+m_2)v$

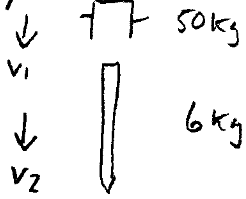
$50\times 2.8 + 0 = 56v$

$v = \frac{50\times 2.8}{56}$

$v = 2.5 \text{ ms}^{-1}$

---

iii)



$e = 0.2$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$50\times 2.8 + 0 = 50v_1 + 6v_2$

$140 = 50v_1 + 6v_2$ (1)

Also $\frac{v_S}{v_A} = e$ $\quad \frac{v_2 - v_1}{2.8} = 0.2$

$v_2 - v_1 = 0.56$

$\Rightarrow 6v_2 - 6v_1 = 3.36$ (2)

(1) − (2)

$56v_1 = 136.64$

pile driver $\underline{v_1 = 2.44 \text{ ms}^{-1}}$

$v_2 = 0.56 + 2.44$

post $\underline{v_2 = 3.0 \text{ ms}^{-1}}$

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iv) ↓ Force on post $= 6g - 600$ N

$ma = 6g - 600$

$a = \frac{6g - 600}{6} = -90.2 \text{ ms}^{-2}$

Find time until rest

$v = u + at$

$0 = 3 - 90.2t$

$t = \frac{3}{90.2} = 0.033 \text{ s}$

19 iv) cont) Distance travelled by post

$$s = ut + \tfrac{1}{2}at^2$$

$$s = 3 \times 0.033 - \tfrac{1}{2} \times 90.2 \times 0.033^2$$

$$s = 0.050 \text{ m}$$

For pile driver $a = 9.8$

Distance travelled in 0.033s

$$s = 2.44 \times 0.033 + \tfrac{1}{2} \times 9.8 \times 0.033^2$$

$$s = 0.086 \text{ m}$$

Since pile driver would travel further than post in time it takes for post to come to rest, they would collide again before this happened.