1)

Outcome	1	2	3	4	5
Probability	0.1	0.2	0.4	0.2	0.1

$E(X) = 1\times0.1 + 2\times0.2 + 3\times0.4 + 4\times0.2 + 5\times0.1$

$= 3$

Could see answer by symmetry of distribution

2)

$P(X=r) = \frac{2r-1}{16}$ for $r = 1,2,3,4$

$P(X=r) = 0$ otherwise

i)

r	1	2	3	4
P(X=r)	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$

$E(X) = \frac{1}{16}[1\times1 + 2\times3 + 3\times5 + 4\times7]$

$= \frac{50}{16} = \frac{25}{8} = \mu$

ii) $P(X<\mu) = \frac{1}{16} + \frac{3}{16} + \frac{5}{16} = \frac{9}{16}$

3)

i)

r	4	5
P(X=r)	p	1−p

$E(X) = 4.2 = 4p + 5(1-p)$

$4.2 = 4p + 5 - 5p$

$4.2 - 5 = -p$

$\Rightarrow p = 0.8$

r	4	5
P(X=r)	0.8	0.2

ii)

r	50	100
P(Y=r)	p	(1−p)

$E(Y) = 80 = 50p + 100(1-p)$

$80 = 50p + 100 - 100p$

$-20 = -50p$

$\Rightarrow p = 0.4$

r	50	100
P(Y=r)	0.4	0.6

4) i)

r	2	3	4	5	6	7	8	9	10	11	12
P(X=r)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

By symmetry $E(X) = 7$

$E(X) = \frac{1}{36}[1\times2 + 2\times3 + 3\times4 + 4\times5 + 5\times6 + 6\times7 + 5\times8 + 4\times9 + 3\times10 + 2\times11 + 1\times12]$

$= \frac{252}{36} = 7$

ii) $Var(X) = E(X^2) - (E(X))^2$

r^2	4	9	16	25	36	49	64	81	100	121	144

$E(X^2) = \frac{1}{36}[1\times4 + 2\times9 + 3\times16 + 4\times25 + 5\times36 + 6\times49 + 5\times64 + 4\times81 + 3\times100 + 2\times121 + 1\times144]$

$= \frac{1974}{36} = 54.833$

$Var(X) = 54.833 - 49$

$= 5.833$

4iii) a) $P(X<\mu) = \frac{1}{36}[1+2+3+4+5]$

$= \frac{15}{36} = \frac{5}{12}$

b) $\sigma = \sqrt{5.833} = 2.415$

$P(X>\mu+\sigma) = P(X>7+2.415)$

$= \frac{1}{36}[3+2+1]$

$= \frac{6}{36} = \frac{1}{6}$

c) $P(|X-\mu|<2\sigma)$

$P(|X-7|<4.83)$

$P(3 \leq X \leq 11)$

$= \frac{34}{36} = \frac{17}{18}$

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5 i)

–	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

r	0	1	2	3	4	5
$P(X=r)$	$\frac{6}{36}$	$\frac{10}{36}$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{2}{36}$

$P(X=r)$	$\frac{3}{18}$	$\frac{5}{18}$	$\frac{4}{18}$	$\frac{3}{18}$	$\frac{2}{18}$	$\frac{1}{18}$

$E(X) = \frac{1}{18}[3\times0 + 5\times1 + 4\times2 + 3\times3 + 2\times4 + 1\times5]$

$= \frac{35}{18} = 1.944$

r^2	0	1	4	9	16	25

$E(X^2) = \frac{1}{18}[3\times0 + 5\times1 + 4\times4 + 3\times9 + 2\times16 + 1\times25]$

$= \frac{105}{18}$

$Var(X) = E(X^2) - (E(X))^2$

$= \frac{105}{18} - \left(\frac{35}{18}\right)^2$

$= 2.052$

ii)

$P(Y>\mu) = P(Y>1.944)$

$= \frac{1}{18}[4+3+2+1] = \frac{10}{18}$

$= \frac{5}{9}$

$\sigma = \sqrt{2.052} = 1.432$

$P(Y>\mu+2\sigma)$

$= P(Y>1.944+2\times1.432)$

$= P(Y>4.808) = \frac{1}{18}$

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6) i)

r	0	1	2	3
$P(X=r)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$E(X) = \frac{1}{8}[1\times0 + 3\times1 + 3\times2 + 1\times3]$

6i) Cont $E(X) = \frac{12}{8} = 1.5$

Also $= 3 \times \frac{1}{2} = 1.5$

ii)

r^2	0	1	4	9
$P(X=r)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$E(X^2) = \frac{1}{8}[1\times0 + 3\times1 + 3\times4 + 1\times9]$

$= \frac{24}{8} = 3$

$Var(X) = E(X^2) - (E(X))^2$

$= 3 - 1.5^2 = 0.75$

Also $= 3 \times \frac{1}{4} = 0.75$

iii)

10 coins $E(Y) = 10 \times \frac{1}{2} = 5$

$Var(Y) = 10 \times \frac{1}{4} = 2.5$

7) i)

r	-5	0	2
$P(X=r)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

ii) $E(X) = -\frac{5}{4} + 0 + 1 = -\frac{1}{4}$

$E(X) = -25$ pence

$E(X^2) = \frac{1}{4} \times 25 + 0 + \frac{1}{2} \times 4$

$= \frac{25}{4} + 2 = 8.25$

$Var(X) = 8.25 - \left(-\frac{1}{4}\right)^2$

$= 8.1875$

$E(X) < 0$ indicates expected loss

iii)

Needs to pay £1 less

8) i)

	warm	normal	coal
r	55	25	5
$P(X=r)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{3}{10}$

ii) $E(X) = \frac{1}{10}[3\times55 + 4\times25 + 3\times5]$

$= \frac{280}{10} = £28$

$E(X^2) = \frac{1}{10}[3\times55^2 + 4\times25^2 + 3\times5^2]$

$= \frac{11650}{10} = 1165$

$Var(X) = E(X^2) - (E(X))^2$

$= 1165 - 28^2$

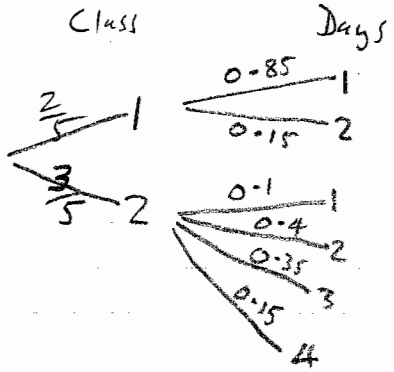
$= 381$

iii)

$E(X) = 28 - 5 = £23$

Var(X) unchanged at 381

9)



$P(X=1) = \frac{2}{5} \times 0.85 + \frac{3}{5} \times 0.1$

$= 0.4$

$P(X=2) = \frac{2}{5} \times 0.15 + \frac{3}{5} \times 0.4$

$= 0.3$

$P(X=3) = \frac{3}{5} \times 0.35 = 0.21$

$P(X=4) = \frac{3}{5} \times 0.15 = 0.09$

ii)

r	1	2	3	4
P(X=r)	0.4	0.3	0.21	0.09

iii) $E(X) = 0.4 \times 1 + 0.3 \times 2 + 0.21 \times 3 + 0.09 \times 4$

$E(X) = 1.99$

$E(X^2) = 0.4 \times 1^2 + 0.3 \times 2^2 + 0.21 \times 3^2 + 0.09 \times 4^2$

$= 4.93$

$Var(X) = E(X^2) - (E(X))^2$

$= 4.93 - 1.99^2 = 0.9699$

10)

r	3	4	5
P(X=r)	p	q	1-p-q

$E(X) = 4 \quad Var(X) = 0.6$

$3p + 4q + 5(1-p-q) = 4$

$3p + 4q + 5 - 5p - 5q = 4$

$-2p - q = -1$

$2p + q = 1 \quad (1)$

$Var(X) = E(X^2) - (E(X))^2$

$0.6 = E(X^2) - 16$

$16.6 = E(X^2)$

$16.6 = 9p + 16q + 25(1-p-q)$

$16.6 = 9p + 16q + 25 - 25p - 25q$

$16.6 = -16p - 9q + 25$

$16p + 9q = 8.4 \quad (2)$

$(1) \times 9 \quad 18p + 9q = 9 \quad (3)$

$(3) - (2) \quad 2p = 0.6$

$p = 0.3$

Subst for p in (1)

$0.6 + q = 1 \Rightarrow q = 0.4$

Giving

r	3	4	5
P(X=r)	0.3	0.4	0.3

10ii)

r	20	50	100
$P(Y=r)$	p	q	$1-p-q$

$E(Y) = 34$ $\quad Var(Y) = 624$

$E(Y) = 34 = 20p + 50q + 100(1-p-q)$

$34 = 20p + 50q + 100 - 100p - 100q$

$34 = -80p - 50q + 100$

$80p + 50q = 66$ (1)

$Var(Y) = E(Y^2) - (E(Y))^2$

$624 = E(Y^2) - 34^2$

$624 = E(Y^2) - 1156$

$1780 = E(Y^2)$

$1780 = 400p + 2500q + 10000(1-p-q)$

$1780 = 400p + 2500q + 10000 - 10000p - 10000q$

$1780 = -9600p - 7500q + 10000$

$9600p + 7500q = 8220$ (2)

(1) × 150

$12000p + 7500q = 9900$ (3)

(3) − (2)

$2400p = 1680$

$p = 0.7$

Subst in (1)

$56 + 50q = 66$

$50q = 10$

$q = 0.2$

Giving

r	20	50	100
$P(Y=r)$	0.7	0.2	0.1